Dawson College: Fall 2019: Linear Algebra (Science): 201-NYC-05-S5: **Quiz 13** name: ________________________________

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Let $V = \{(a,b,c) + s\vec{d_1} + t\vec{d_2} \mid s,t \in \mathbb{R}\}$ for fixed (a,b,c), $\vec{d_1}$ and $\vec{d_2}$ in $\mathbb{R}^3$. In addition, $\vec{d_1}$ and $\vec{d_2}$ are not parallel nor the zero vector.

a. *(1 mark)* Give a geometric description of V, include a sketch of V.

b. *(5 marks)* Show that V with the standard operations of $\mathbb{R}^3$ is a vector space if and only if $(a,b,c) = (0,0,0)$.

Question 2. *(3 marks)* Determine whether the following is a subspace of $F(-\infty,\infty)$: All functions f in $F(-\infty,\infty)$ for which $f(0) = 1$.

Question 3. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement. Two subsets of a vector space V that span the same subspace of V must be equal.

Question 4. *(2 marks)* Is the relation 'is a subspace of' transitive? That is, if V is a subspace of W and W is a subspace of X, must V be a subspace of X?

Question 5. *(1 mark)* Let $\mathbf{u}_4$ be a vector that is not a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \}$.
Select the best statement.

1. There is no obvious relationship between span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$.
2. span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ = span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$.
3. We only know that span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\} \subseteq$ span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
4. span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ = span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ when none of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a linear combination of the others.
5. We only know that span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} \subseteq$ span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$.
6. span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a proper subset of span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$.
7. none of the above

Question 6. *(1 mark)*
Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ spans $\mathbb{R}^3$.
Select the best statement.

1. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is the zero vector.
2. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans $\mathbb{R}^3$.
3. There is no easy way to determine if $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$.
4. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is a scalar multiple of another vector in the set.
5. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans $\mathbb{R}^3$.
6. none of the above

Question 7. *(1 mark)*
Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ does not span $\mathbb{R}^3$.
Select the best statement.

1. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ may, but does not have to, span $\mathbb{R}^3$.
2. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is a scalar multiple of another vector in the set.
3. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is the zero vector.
4. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans $\mathbb{R}^3$.
5. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans $\mathbb{R}^3$.
6. none of the above