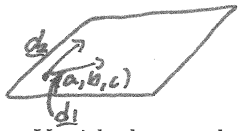
Dawson College: Fall 2019: Linear Algebra (SCIENCE): 201-NYC-05-S5: **Quiz 13** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Let $V = \{(a,b,c) + s\vec{d_1} + t\vec{d_2} \mid s,t \in \mathbb{R}\}$ for fixed (a,b,c), $\vec{d_1}$ and $\vec{d_2}$ in $\mathbb{R}^3$. and d_1, d_2 are not parallel nor zero

a. *(1 mark)* Give a geometric description of V, include a sketch of V.

V is a plane in $\mathbb{R}^3$ that contains the point (a,b,c) and parallel to both $\underline{d_1}$ and $\underline{d_2}$.



b. *(5 marks)* Show that V with the standard operations of $\mathbb{R}^3$ is a vector space if and only if $(a,b,c) = (0,0,0)$.

[$\Rightarrow$] Suppose $(a,b,c) \neq \underline{0}$. Let $\underline{u}, \underline{v} \in V$ then $\underline{u} = (a,b,c) + s_1\underline{d_1} + t_1\underline{d_2}$
$\underline{v} = (a,b,c) + s_2\underline{d_1} + t_2\underline{d_2}$

and $\underline{u}+\underline{v} = 2(a,b,c) + (s_1+s_2)\underline{d_1} + (t_1+t_2)\underline{d_2} \notin V$ $\therefore$ not closed under addition $\therefore$ contradicts the fact that V is a vector space $\therefore (a,b,c) = \underline{0}$

[$\Leftarrow$] $V = \{s\underline{d_1} + t\underline{d_2} \mid s,t \in \mathbb{R}\}$. Lets apply the subspace test.

<u>closure under addition:</u> Let $\underline{u}, \underline{v} \in V$ then $\underline{u} = s_1\underline{d_1} + t_1\underline{d_2}$
$\underline{v} = s_2\underline{d_1} + t_2\underline{d_2}$

and $\underline{u} + \underline{v} = (s_1+s_2)\underline{d_1} + (t_1+t_2)\underline{d_2} \in V$

<u>closure under scalar mult.</u> Let $\underline{u} \in V$ and $r \in \mathbb{R}$ then $\underline{u} = s_1 d_1 + t_1\underline{d_2}$

$r\underline{u} = r(s_1\underline{d_1} + t_1\underline{d_2}) = rs_1 d_1 + rt_1 \underline{d_2} \in V$

$\therefore$ by the subspace test, V is a subspace of $\mathbb{R}^3$

$\therefore$ V is a vector space

Question 2. *(3 marks)* Determine whether the following is a subspace of $F(-\infty,\infty)$: All functions f in $F(-\infty,\infty)$ for which $f(0) = 1$.

Not a subspace since the set is not closed under addition.

Let $f, g \in W$ where $W = \{f \mid f: \mathbb{R} \to \mathbb{R} \text{ and } f(0) = 1\}$

then $f(0) = 1$ and $g(0) = 1$.

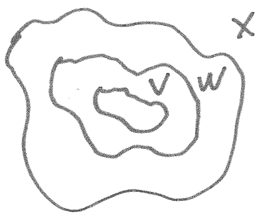
$f+g \notin W$ since $(f+g)(0) = f(0) + g(0) = 1+1 = 2 \neq 1$.

Question 3. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement. Two subsets of a vector space V that span the same subspace of V must be equal.

False,

$\text{span}(\{(1,1)\}) = \text{span}(\{(2,2)\})$ but $\{(1,1)\} \neq \{(2,2)\}$

Question 4. *(2 marks)* Is the relation 'is a subspace of' transitive? That is, if V is a subspace of W and W is a subspace of X, must V be a subspace of X?



Since V is a subspace of W, we have $V \subseteq W$
" W " " " X " " $W \subseteq X$

$\therefore V \subseteq X$ and the operations are the same for all spaces. By the subspace test we have that V is a subspace of X.

Question 5. *(1 mark)* Let $\mathbf{u}_4$ be a vector that is not a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \}$.
Select the best statement.

1. There is no obvious relationship between span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$.
2. span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} =$ span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$.
3. We only know that span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\} \subseteq$ span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
4. span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} =$ span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ when none of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a linear combination of the others.
5. We only know that span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} \subseteq$ span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$.
6. span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a proper subset of span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$.
7. none of the above

Question 6. *(1 mark)*
Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ spans $\mathbb{R}^3$.
Select the best statement.

1. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is the zero vector.
2. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans $\mathbb{R}^3$.
3. There is no easy way to determine if $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$.
4. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is a scalar multiple of another vector in the set.
5. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans $\mathbb{R}^3$.
6. none of the above

Question 7. *(1 mark)*
Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ does not span $\mathbb{R}^3$.
Select the best statement.

1. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ may, but does not have to, span $\mathbb{R}^3$.
2. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is a scalar multiple of another vector in the set.
3. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is the zero vector.
4. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans $\mathbb{R}^3$.
5. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans $\mathbb{R}^3$.
6. none of the above