Dawson College: Fall 2019: Linear Algebra (Science): 201-NYC-05-S5: **Quiz 14** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. (4 *marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement. Show all your work!

a. There is a basis for M_{22} consisting of invertible matrices. $\boxed{\text{True}}$

Let $M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $M_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $M_3 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $M_4 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, they are all invertible since they are elementary matrices. Consider $c_1M_1 + c_2M_2 + c_3M_3 + c_4M_4 = 0$, we obtain

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}}_{\underline{c}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\underline{0}}$$

and since
$$|A| = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$
$$= \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - [1]$$
$$= -1 + 1 - [1]$$
$$= -1 \neq 0$$

$\therefore$ only trivial solution by equivalence thm

$\therefore$ $S = \{M_1, M_2, M_3, M_4\}$ is l.i.

$\therefore$ S also spans $M_{2\times 2}$ since S is l.i. has 4 vectors which is equal to $\dim(M_{2\times 2}) = 4$. $\therefore$ S is a basis of $M_{2\times 2}$.

b. If A has size $n \times n$ and $I_n, A, A^2, \ldots, A^{n^2}$ are distinct matrices, then $\underbrace{\{I_n, A, A^2, \ldots, A^{n^2}\}}_{S}$ is a linearly dependent set.

True,
Since the number of vectors in S is $n^2+1 > \dim(M_{n\times n}) = n^2$ then by a corollary seen in class the set S must be lin. dep.

c. There are only three distinct two-dimensional subspaces of P_2.

False,

The space with basis $\{1, x\}$
" " " " $\{1, x^2\}$
" " " " $\{x, x^2\}$
" " " " $\{1+x, x^2\}$

All are lin. ind. set since in each set no vector is a mult. of the other. Clearly they are distinct, the space they generate are different.

d. Every linearly independent subset of a vector space V is a basis for V.

False, Let $V = \mathbb{R}^2$, $S = \{(1,0)\}$ is lin. ind. but is not a basis for V since it only spans the vectors along the x-axis.