Dawson College: Fall 2019: Linear Algebra (SCIENCE): 201-NYC-05-S5: **Quiz 2** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 2.[1] *(6 marks)* Consider the following system:

$$\begin{array}{rcrcrcl} 2x & + & 3y & + & 4z & = & 3 \\ 2x & + & (h+1)y & + & 6z & = & 4 \\ 4x & + & 6y & + & (h-4)z & = & k-1 \end{array}$$

$$\begin{bmatrix} 2 & 3 & 4 & 3 \\ 2 & h+1 & 6 & 4 \\ 4 & 6 & h-4 & k-1 \end{bmatrix} \underset{-2R_1+R_3\to R_3}{\overset{-R_1+R_2\to R_2}{\sim}} \begin{bmatrix} 2 & 3 & 4 & 3 \\ 0 & h-2 & 2 & 1 \\ 0 & 0 & h-12 & k-7 \end{bmatrix}$$

Find all value(s) of h and k, if possible, such that the system has: a unique solution, infinitely many solutions, no solution. *Be very clear about your combinations of h and k in each case and justify!*

a unique solution if # leading entries in var. col. = # var. So $h-2\neq 0$ and $h-12\neq 0$.

$\therefore$ unique solution if $h\neq 2$ and $h\neq 12$. And $k\in\mathbb{R}$

infinitely many solutions if # leading entries in var. col. < # var. So case 1) $h=2$ then the augmented matrix becomes

$$\begin{bmatrix} 2 & 3 & 4 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -10 & k-7 \end{bmatrix} \underset{5R_2+R_3\to R_3}{\sim} \begin{bmatrix} 2 & 3 & 4 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & k-2 \end{bmatrix}$$

If $k=2$ then ∞ many sol. If $k\neq 2$ then no solution since there is a leading entry in the constant column.

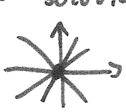
case 2) $h=12$ then the augmented matrix becomes

$$\begin{bmatrix} 2 & 3 & 4 & 3 \\ 0 & 10 & 2 & 1 \\ 0 & 0 & 0 & k-7 \end{bmatrix}$$

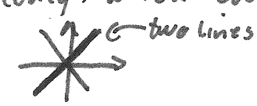
If $k=7$ then ∞ many sol.
If $k\neq 7$ then no solutions

Question 2. *(2 marks)* Discuss the relative positions of the lines $ax+by=0$, $cx+dy=0$, and $ex+fy=0$ when the system composed of the three lines has only the trivial solution and when it has nontrivial solutions.

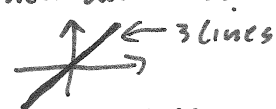
Trivial solution (only) when at least one line has a different slope than the others. i.e.



or



Nontrivial solutions when all lines have the same slope. i.e.



Question 3. Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(2 marks)* If an elementary row operation is applied to a matrix that is in row echelon form, the resulting matrix will still be in row echelon form.

False, $\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}}_{\text{matrix in REF}} \underset{R_1+R_2\to R_2}{\sim} \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}_{\text{not in REF}}$

b. *(2 marks)* Every matrix has a unique row echelon form.

False, $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \underset{R_2+R_1\to R_1}{\sim} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ Two REF of the same matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

c. *(2 marks)* If a linear system has more unknowns than equations, then it must have infinitely many solutions.

False, $\begin{array}{l} x+y+z=1 \\ x+y+z=2 \end{array}$ less variables than equations but has no solutions since a set of numbers can not add to 1 and 2

[1] From a previous John Abbott final examination