Dawson College: Fall 2019: Linear Algebra (SCIENCE): 201-NYC-05-S5: **Quiz 3** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(2 marks)* A matrix B is said to be a square root of a matrix A if $BB = A$. Find two square roots of $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

If $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ then $BB = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ $\therefore B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is a square root of A.

If $B = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$ then $BB = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ $\therefore B = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$ is a square root of A.

Question 2. Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(3 marks)* If A is a square matrix matrix, then $\mathrm{tr}(A + A^T) = 2\mathrm{tr}(A)$.

True, Suppose $A = [a_{ij}]_{n\times n}$, $tr(A+A^T) = tr([a_{ij}] + [a_{ji}]) = tr([a_{ij} + a_{ji}])$

$= a_{11} + a_{11} + a_{22} + a_{22} + \cdots + a_{nn} + a_{nn} = 2(a_{11} + a_{22} + \cdots + a_{nn}) = 2tr(A)$

b. *(3 marks)* If A and B are square matrices of the same order, then $\mathrm{tr}(AB) = \mathrm{tr}(A)\mathrm{tr}(B)$.

False, Suppose $A = B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ then $AB = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ and $tr(AB) = 5+5 = 10$

But $tr(A)tr(B) = (1+1)(1+1) = 4$

c. *(3 marks)* If B has a column of zeros, then so does AB if this product is defined.

True, Suppose the j^{th} column of B has only zeros then

[j^{th} column of AB] $= A$ [j^{th} column of B]

$= A \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

d. *(3 marks)* If $AB + BA$ is defined, then A and B are square matrices of the same size.

True, Suppose A is an $m\times n$ matrix
and B " " $p\times q$ " .

Then the product AB and BA are $m\times q$ and $p\times n$ matrices, respectively. Since the sum is defined, AB and BA are of the same size, therefore $m=p$ and $n=q$. So we can conclude that A and B have the same dimension i.e. A and B are $m\times n$ matrices. In addition, since the product AB is defined, # col A = # rows of B, therefore $n = m$.

$\therefore$ A and B are $n\times n$ matrices.