

**Question 1.**<sup>1</sup> (3 marks) Given  $A = \begin{bmatrix} -2 & 4 \\ 6 & -7 \end{bmatrix}$ , find a matrix  $X$  such that  $XA - XA^T = A$ .

**Question 2.**<sup>1</sup> (3 marks) Consider the matrix equation  $A^{-1}B = (C - 2A)^{-1}$ . Solve for  $A$ .

**Question 3.** Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. (2 marks) For all square matrices  $A$  and  $B$  of the same size, it is true that  $(A + B)^2 = A^2 + 2AB + B^2$ .

b. (2 marks) The sum of two invertible matrices of the same size must be invertible.

**Question 4.** (3 marks) Prove: If  $A$  has two rows which are identical then  $A$  is singular.

**Bonus.** (3 marks) Find the formula for the  $n$ -th power of this matrix.  $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

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<sup>1</sup>From a past John Abbott final examination