

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.**<sup>1</sup> (3 marks) Show that if  $A$  and  $B$  are square matrices such that  $AB$  is invertible, then  $A$  can be written as a product of elementary matrices.

**Question 2.**<sup>1</sup> Given the matrix  $A = \begin{pmatrix} 0 & 1 & 0 \\ 5 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ .

a. (3 marks) Find  $A^{-1}$ .

b. (2 marks) Express  $A^{-1}$  as a product of elementary matrices, if possible. **Justify your work!**

c. (2 marks) Express  $A$  as a product of elementary matrices, if possible. **Justify your work!**

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<sup>1</sup>From a past Dawson final examination

**Question 3.** Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. (3 marks) If  $A$  and  $B$  are row equivalent matrices, then the linear systems  $Ax = 0$  and  $Bx = 0$  have the same solution set.

b. (3 marks) Let  $A$  be an  $n \times n$  matrix and  $S$  is an  $n \times n$  invertible matrix. If  $\mathbf{x}$  is a solution to the linear system  $(S^{-1}AS)\mathbf{x} = \mathbf{b}$ , then  $S\mathbf{x}$  is a solution to the linear system  $A\mathbf{y} = S\mathbf{b}$ .

c. (3 marks) Every elementary matrix is invertible.

**Bonus.** (3 marks) Find the formula for the  $n$ -th power of this matrix.  $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$