Dawson College: Fall 2019: Linear Algebra (SCIENCE): 201-NYC-05-S5: **Quiz 6** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(3 marks)* Show that $A^T(4A)$ must be symmetric. **Justify your work completely, do not skip steps!**

Must show that $(A^T(4A))^T = A^T(4A)$.

$$\begin{aligned} LHS &= (A^T(4A))^T \\ &= (4A)^T (A^T)^T \\ &= 4A^T A \\ &= A^T 4A \\ &= A^T(4A) = RHS \end{aligned}$$

Question 2.[2] *(5 marks)* Solve for x where

$$\begin{vmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ e^x & 1 & e^x \\ 1 & e^x & 0 \end{vmatrix}$$

$$\sin^2 + \cos^2 x = a_{31}C_{31} + a_{32}C_{32} + \underbrace{a_{33}C_{33}}_{0}$$

$$1 = 1\begin{vmatrix} 1 & 1 \\ 1 & e^x \end{vmatrix} - e^x \begin{vmatrix} 1 & 1 \\ e^x & e^x \end{vmatrix}$$

$$1 = e^x - 1$$

$$2 = e^x$$

$$x = \ln 2$$

Question 3. Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(2 marks)* If A^2 is a symmetric matrix, then A is a symmetric matrix.

False, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is not symmetric but A^2 is symmetric since $A^2 = 0$

b. *(2 marks)* If A is a square matrix whose minors are all zero, then $\det(A) = 0$.

True,
$$\begin{aligned} \det(A) &= a_{11}C_{11} + a_{12}C_{12} + \cdots + a_{1n}C_{1n} \\ &= a_{11}M_{11} + a_{12}(-M_{12}) + \cdots + a_{1n}(-1)^{1+n}M_{1n} \\ &= a_{11}(0) + a_{12}(-0) + \cdots + a_{1n}(-1)^{1+n}(0) \\ &= 0 \end{aligned}$$

Question 4. *(3 marks)* A matrix A is said to be *skew-symmetric* if $A^T = -A$. If A is an invertible skew-symmetric matrix, then A^{-1} is skew-symmetric.

Must show $(A^{-1})^T = -A^{-1}$.

$$\begin{aligned} LHS &= (A^{-1})^T \\ &= (A^T)^{-1} && \text{by thm seen in class} \\ &= (-A)^{-1} && \text{by premise} \\ &= \frac{1}{-1}A^{-1} && \text{by thm seen in class} \\ &= -A^{-1} \\ &= RHS \end{aligned}$$

[1]From a past John Abbott final examination

[2]From a past Dawson College final examination