Dawson College: Fall 2019: Linear Algebra (SCIENCE): 201-NYC-05-S5: **Quiz 7** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(3 marks)* Complete the following sentences with the word **must**, **might** or, **cannot**, as appropriate.

a. If R is the reduced row echelon form of a matrix which is row equivalent to an invertible matrix, then $\det(R)$ Cannot equal zero.

b. If A, B, and C are square matrices such that $ABC^2 = I$ then matrix must be invertible.

c. Let A be a square matrix. If $A\mathbf{x} = A\mathbf{y}$ for two distinct $\mathbf{x}$ and $\mathbf{y}$ then $\det(A)$ must equal zero.

Question 2.[2] *(4 marks)* Let A and B be two 3×3 matrices with $\det(A) = 2$ and $\det(B) = 5$. Find: $\det(A^3 \det(B) A^T\ (\text{adj}(A))^4)$.

$$= \det(A^3)\det(\det(B)A^T)\det((\text{adj}(A))^4)$$
$$= (\det(A))^3(\det(B))^3\det(A^T)(\det(\text{adj}(A)))^4$$
$$= 2^3 5^3 \det(A)((\det(A))^{3-1})^4$$
$$= 2^3 5^3\, 2\,((2)^2)^4$$
$$= 2^3 5^3\, 2\,(2^8)$$
$$= 2^{12} 5^3$$

Question 3.[2] *(3 marks)* If A and B are invertible matrices of the same size show that $\text{adj}(AB) = \text{adj}(B)\,\text{adj}(A)$.

Since A and B are invertible it follows that $A^{-1} = \frac{1}{\det A}\text{adj} A$, $B^{-1} = \frac{1}{\det B}\text{adj} B$

and AB is invertible. So $(AB)^{-1} = \frac{1}{\det(AB)}\text{adj}(AB)$

$$\det(AB)(AB)^{-1} = \text{adj}(AB)$$
$$\det(A)\det(B)B^{-1}A^{-1} = \text{adj}(AB)$$
$$\det(B)B^{-1}\det(A)A^{-1} = \text{adj}(AB)$$

$$\text{adj}(AB) = \det(B)\frac{1}{\det(B)}\text{adj}(B)\,\det(A)\frac{1}{\det(A)}\text{adj}(A)$$
$$\text{adj}(AB) = \text{adj}(B)\,\text{adj}(A)$$

Question 4.[1] *(4 marks)* Given that $\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = 10$, find $\begin{vmatrix} 3g+a & 3h+b & 2 & 3i+c \\ d+2a & e+2b & 3 & f+2c \\ a & b & 4 & c \\ 0 & 0 & 5 & 0 \end{vmatrix}$

$$= \underbrace{a_{41}}_{0}C_{41} + \underbrace{a_{42}}_{0}C_{42} + a_{43}C_{43} + \underbrace{a_{44}}_{0}C_{44}$$
$$= a_{43}C_{43}$$
$$= 5(-1)^{4+3}\begin{vmatrix} 3g+a & 3h+b & 3i+c \\ d+2a & e+2b & f+2c \\ a & b & c \end{vmatrix}$$
$$= -5 \underset{-2R_3+R_2 \to R_2}{-R_3+R_1 \to R_1} \begin{vmatrix} 3g & 3h & 3i \\ d & e & f \\ a & b & c \end{vmatrix}$$
$$= (-5)(-1)\ R_1 \leftrightarrow R_3 \begin{vmatrix} a & b & c \\ d & e & f \\ 3g & 3h & 3i \end{vmatrix}$$
$$= 5(3)\ \tfrac{1}{3}R_3 \to R_3 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$
$$= 5(3)(10)$$
$$= 150$$

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[1] Inspired from John Abbott Final Examinations.

[2] From a past Dawson College final examination

Question 5. Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(2 marks)* If E is an elementary matrix, then $E\mathbf{x} = \mathbf{0}$ has only the trivial solution. True,

Since E is invertible $E\underline{x} = \underline{0}$ has only the trivial solution by the equivalence theorem.

Bonus. *(5 marks)* Prove: For every $n \times n$ matrix A, we have $A\text{adj}(A) = (\det(A))I_n$.