Dawson College: Fall 2019: Linear Algebra (SCIENCE): 201-NYC-05-S5: **Quiz 8** name: _____

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Given two points A and B in $\mathbb{R}^n$. Using vectors find the formula for the midpoint of the line segment connecting the points A and B. That is, show that the midpoint is $\frac{1}{2}(A+B)$.

Question 2.[1] *(4 marks)* Find a unit vector that bisects the smaller of the two angles formed by the vectors $(3,4)$ and $(5,12)$.

Question 3. *(2 marks)* Given $A(1,2,3)$ and $B(4,3,5)$. Sketch an equivalent vector to $\vec{AB}$ where its initial point is positioned at the origin. In the sketch include the axes and their labels as shown in class.

Question 4. *(3 marks)* Prove: If $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^n$ then $\vec{u}\cdot\vec{v} = \frac{1}{4}||\vec{u}+\vec{v}||^2 - \frac{1}{4}||\vec{u}-\vec{v}||^2$.

[1] Inspired from a WebWork problem

Question 5. Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(2 marks)* In $\mathbb{R}^2$, the vectors of norm 5 whose initial points are at the origin have terminal points lying on a circle of radius 5 centered at the origin.

b. *(2 marks)* If $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w}$, then $\vec{v} = \vec{w}$

Bonus. *(3 marks)* Prove that the quadrilateral $PQRS$, whose vertices are the midpoints of the sides of an arbitrary quadrilateral $ABCD$, is a parallelogram.