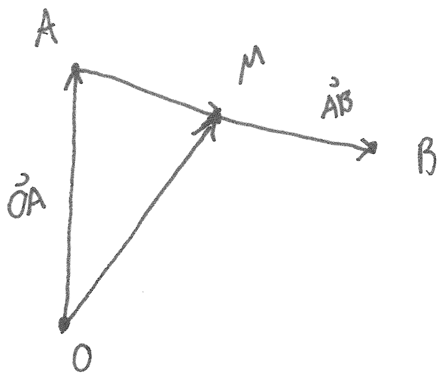
Dawson College: Fall 2019: Linear Algebra (SCIENCE): 201-NYC-05-S5: **Quiz 8** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

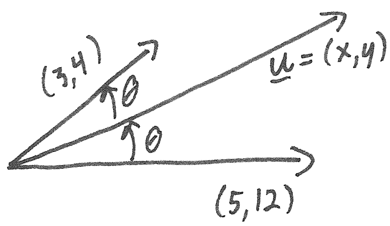
Question 1. *(4 marks)* Given two points A and B in $\mathbb{R}^n$. Find the formula for the midpoint of the line segment connecting the points A and B using vectors. That is, show that the midpoint is $\frac{1}{2}(A+B)$.



$$\vec{OM} = \vec{OA} + \vec{AM}$$
$$= \vec{OA} + \frac{1}{2}\vec{AB}$$
$$= \vec{OA} + \frac{1}{2}(\vec{OB} - \vec{OA})$$
$$= \frac{1}{2}(\vec{OA} + \vec{OB})$$

$$\therefore M = \frac{1}{2}(A+B)$$

Question 2.[1] *(4 marks)* Find a unit vector that bisects the smaller of the two angles formed by the vectors $(3,4)$ and $(5,12)$.



① $\underline{u} \cdot (3,4) = \|\underline{u}\| \|(3,4)\| \cos\theta$

② $\underline{u} \cdot (5,12) = \|\underline{u}\| \|(5,12)\| \cos\theta$

From ① and ② we get

$$\frac{\underline{u} \cdot (3,4)}{\|(3,4)\|} = \|\underline{u}\| \cos\theta$$

$$\frac{\underline{u} \cdot (5,12)}{\|(5,12)\|} = \|\underline{u}\| \cos\theta$$

$$\frac{\underline{u} \cdot (3,4)}{\|(3,4)\|} = \frac{\underline{u} \cdot (5,12)}{\|(5,12)\|}$$

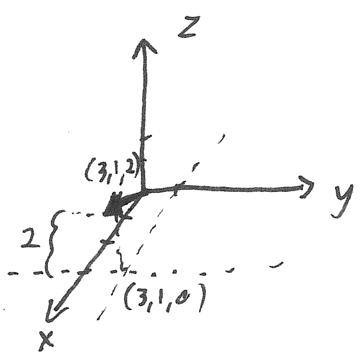
$$\frac{3x+4y}{5} = \frac{5x+12y}{13}$$

$$(3x+4y)13 = 5(5x+12y)$$
$$39x + 52y = 25x + 60y$$
$$14x = 8y$$

Let $x = 4$ then $y = 7$

$$\frac{\underline{u}}{\|\underline{u}\|} = \frac{(4,7)}{\|(4,7)\|} = \left(\frac{4}{\sqrt{65}}, \frac{7}{\sqrt{65}}\right)$$

Question 3. *(2 marks)* Sketch the vector $\vec{AB}$ where $A(1,2,3)$ and $B(4,3,5)$ positioned with its initial point at the origin. In the sketch include the axes and their labels as shown in class.



$$\vec{AB} = \vec{OB} - \vec{OA} = (4,3,5) - (1,2,3) = (3,1,2)$$

Question 4. *(3 marks)* Prove: If $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^n$ then $\vec{u} \cdot \vec{v} = \frac{1}{4}\|\vec{u}+\vec{v}\|^2 - \frac{1}{4}\|\vec{u}-\vec{v}\|^2$.

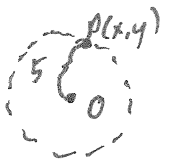
$$RHS = \frac{1}{4}\|\underline{u}+\underline{v}\|^2 - \frac{1}{4}\|\underline{u}-\underline{v}\|^2$$
$$= \frac{1}{4}(\underline{u}+\underline{v})\cdot(\underline{u}+\underline{v}) - \frac{1}{4}(\underline{u}-\underline{v})\cdot(\underline{u}-\underline{v})$$
$$= \frac{1}{4}(\cancel{\underline{u}\cdot\underline{u}} + \underline{u}\cdot\underline{v} + \underline{v}\cdot\underline{u} + \cancel{\underline{v}\cdot\underline{v}}) - \frac{1}{4}(\cancel{\underline{u}\cdot\underline{u}} - \underline{u}\cdot\underline{v} - \underline{v}\cdot\underline{u} + \cancel{\underline{v}\cdot\underline{v}})$$
$$= \frac{1}{4}(2\,\underline{u}\cdot\underline{v}) - \frac{1}{4}(-2\,\underline{u}\cdot\underline{v}) = \underline{u}\cdot\underline{v}$$

[1]Inspired from a WebWork problem

Question 5. Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(2 marks)* In $\mathbb{R}^2$, the vectors of norm 5 whose initial points are at the origin have terminal points lying on a circle of radius 5 centered at the origin.

True,



$$\|\vec{OP}\| = 5$$
$$\|(x,y)\| = 5$$
$$\sqrt{x^2+y^2} = 5$$
$$x^2+y^2 = 5^2$$

b. *(2 marks)* If $\vec{u}\cdot\vec{v} = \vec{u}\cdot\vec{w}$, then $\vec{v} = \vec{w}$

False, if $\underline{u} = (0,0)$ and $\underline{v} = (1,1)$ and $\underline{w} = (1,2)$ we have

$\underline{u}\cdot\underline{v} = 0 = \underline{u}\cdot\underline{w}$ but $\underline{v} \neq \underline{w}$

Bonus. *(3 marks)* Prove that the quadrilateral $PQRS$, whose vertices are the midpoints of the sides of an arbitrary quadrilateral $ABCD$, is a parallelogram.