

Question 1.¹ (1 mark each) Given

$$\mathcal{P}_1 : 2x + y - 3z = 6,$$

$$\mathcal{P}_2 : -6x - 3y + 9z = 1,$$

$$\mathcal{P}_3 : x + y + z = 1, \text{ and}$$

$$\mathcal{L}_1 : \vec{x} = (1, 0, 1) + t(-4, -2, 6) \text{ where } t \in \mathbb{R}.$$

Complete the following sentences with the word **perpendicular**, **parallel** or, **neither perpendicular nor parallel**, as appropriate.

- $\mathcal{P}_1$ and $\mathcal{P}_2$ are _____ to each other.
- $\mathcal{P}_1$ and $\mathcal{P}_3$ are _____ to each other.
- $\mathcal{P}_1$ and $\mathcal{L}_1$ are _____ to each other.
- $\mathcal{P}_3$ and $\mathcal{L}_1$ are _____ to each other.

Question 2. Given the plane $\mathcal{P} : 3x + 2y + z = 6$.

- (2 marks) Find the x , y and z intercept of $\mathcal{P}$ and sketch $\mathcal{P}$, include the axes and their labels as shown in class.

- (4 marks) Using projection(s) find the distance between the origin and $\mathcal{P}$.

- (3 marks) Find the angle between $\mathcal{P}$ and the xz -plane (the plane that contains the x and z axis).

¹ Inspired from John Abbott Final Examinations.

Question 3. (4 marks) Find the closest point on $x - y = 0$ to the point $P(2, 3)$.

Question 4. Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. (2 marks) If $\vec{a}$ and $\vec{b}$ are orthogonal vectors, then for every nonzero vector $\vec{u}$, we have $\text{proj}_{\vec{a}}(\text{proj}_{\vec{b}}(\vec{u})) = \vec{0}$

b. (2 marks) If the relationship $\text{proj}_{\vec{a}}(\vec{u}) = \text{proj}_{\vec{a}}(\vec{v})$ holds for some nonzero vector $\vec{a}$, then $\vec{u} = \vec{v}$.