Dawson College: Fall 2019: Linear Algebra (SCIENCE): 201-NYC-05-S5: **Quiz 9** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

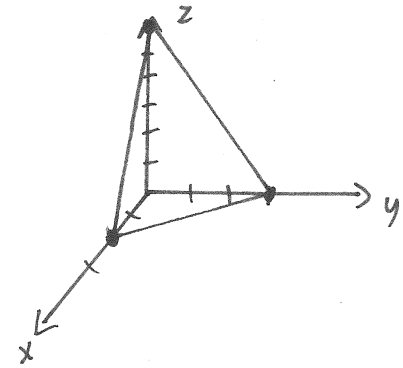
Question 1.[1] *(1 mark each)* Given
$\mathcal{P}_1 : 2x + y - 3z = 6$,
$\mathcal{P}_2 : -6x - 3y + 9z = 1$,
$\mathcal{P}_3 : x + y + z = 1$, and
$\mathcal{L}_1 : \vec{x} = (1, 0, 1) + t(-4, -2, 6)$ where $t \in \mathbb{R}$.
Complete the following sentences with the word **perpendicular**, **parallel** or, **neither perpendicular nor parallel**, as appropriate.

a. $\mathcal{P}_1$ and $\mathcal{P}_2$ are $\parallel$ to each other.
b. $\mathcal{P}_1$ and $\mathcal{P}_3$ are $\perp$ to each other.
c. $\mathcal{P}_1$ and $\mathcal{L}_1$ are $\perp$ to each other.
d. $\mathcal{P}_3$ and $\mathcal{L}_1$ are $\parallel$ to each other.

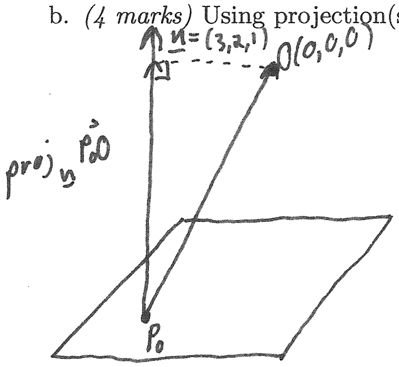
Question 2. Given the plane $\mathcal{P} : 3x + 2y + z = 6$.

a. *(2 marks)* Find the x, y and z intercept of $\mathcal{P}$ and sketch $\mathcal{P}$, include the axes and their labels as shown in class.

x-int: Let $y=z=0 \Rightarrow x=2$ $(2,0,0)$
y-int: Let $x=z=0 \Rightarrow y=3$ $(0,3,0)$
z-int: Let $x=y=0 \Rightarrow z=6$ $(0,0,6)$



b. *(4 marks)* Using projection(s) find the distance between the origin and $\mathcal{P}$.



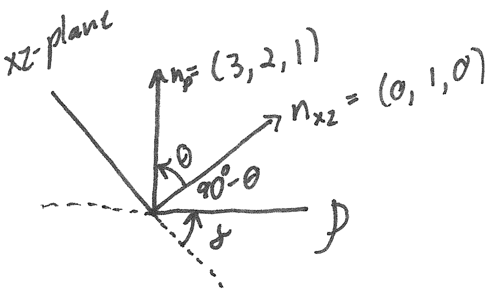
Let $y=z=0 \Rightarrow x=2$ $\therefore P_0(2,0,0)$

$\vec{P_0O} = \vec{O} - \vec{OP_0} = (0,0,0) - (2,0,0) = (-2,0,0)$

$\text{proj}_{\underline{n}} \vec{P_0P} = \dfrac{\vec{P_0P} \cdot \underline{n}}{\underline{n} \cdot \underline{n}} \underline{n} = \dfrac{3(-2) + (2)(0) + 1(0)}{3 \cdot 3 + 2 \cdot 2 + 1 \cdot 1}(3,2,1) = \dfrac{-6}{14}(3,2,1)$
$= \dfrac{-3}{7}(3,2,1) = \left(\dfrac{-9}{7}, \dfrac{-6}{7}, \dfrac{-3}{7}\right)$

distance $= \|\text{proj}_{\underline{n}} \vec{P_0P}\| = \|\dfrac{-3}{7}(3,2,1)\|$
$= \dfrac{3}{7}\|(3,2,1)\| = \dfrac{3}{7}\sqrt{14}$

c. *(3 marks)* Find the angle between $\mathcal{P}$ and the xz-plane (the plane that contains the x and z axis).



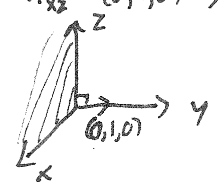
$n_p \cdot n_{xz} = \|n_p\|\|n_z\| \cos\theta$

$2 = \sqrt{14}\cos\theta$

$\dfrac{2}{\sqrt{14}} = \cos\theta$

$\gamma = \theta = \arccos\left(\dfrac{2}{\sqrt{14}}\right)$

xz-plane has normal vector $n_{xz} = (0,1,0)$ since



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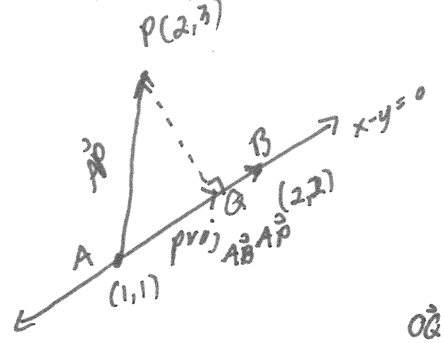
[1] Inspired from John Abbott Final Examinations.

Question 3. *(4 marks)* Find the closest point on $x - y = 0$ to the point $P(2,3)$.



$$\vec{AP} = \vec{OP} - \vec{OA} = (2,3) - (1,1) = (1,2)$$

$$\vec{AB} = \vec{OB} - \vec{OA} = (2,2) - (1,1) = (1,1)$$

$$\vec{AQ} = \text{proj}_{\vec{AB}} \vec{AP}$$

$$\vec{AQ} = \frac{\vec{AP} \cdot \vec{AB}}{\vec{AB} \cdot \vec{AB}} \vec{AB}$$

$$\vec{OQ} - \vec{OA} = \frac{1+2}{2}(1,1)$$

$$\vec{OQ} = (1,1) + \frac{3}{2}(1,1) = \left(\frac{5}{2}, \frac{5}{2}\right)$$

Question 4. Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(2 marks)* If $\vec{a}$ and $\vec{b}$ are orthogonal vectors, then for every nonzero vector $\vec{u}$, we have $\text{proj}_{\vec{a}}(\text{proj}_{\vec{b}}(\vec{u})) = \vec{0}$

True, $\text{proj}_{\underline{a}}(\text{proj}_{\underline{b}}\, \underline{u}) = \text{proj}_{\underline{a}}\left(\frac{\underline{b}\cdot\underline{u}}{\underline{b}\cdot\underline{b}}\underline{b}\right) = \dfrac{\underline{a}\cdot\left(\frac{\underline{b}\cdot\underline{u}}{\underline{b}\cdot\underline{b}}\underline{b}\right)}{\underline{a}\cdot\underline{a}}\underline{a}$

$= \dfrac{\underline{b}\cdot\underline{u}}{\underline{b}\cdot\underline{b}} \dfrac{\underline{a}\cdot\underline{b}}{\underline{a}\cdot\underline{a}}\underline{a} = \dfrac{\underline{b}\cdot\underline{u}}{\underline{b}\cdot\underline{b}} \cdot \dfrac{0}{\underline{a}\cdot\underline{a}}\underline{a}$ since $\underline{a}\cdot\underline{b} = 0$

$= 0$

b. *(2 marks)* If the relationship $\text{proj}_{\vec{a}}(\vec{u}) = \text{proj}_{\vec{a}}(\vec{v})$ holds for some nonzero vector $\vec{a}$, then $\vec{u} = \vec{v}$.

False, Let $\underline{u} = (1,1)$ and $\underline{v} = (1,2)$ and $\underline{a} = (1,0)$

$\text{proj}_{\underline{a}}\underline{u} = \text{proj}_{\underline{a}}\underline{v} = (1,0)$

but $\underline{u} \neq \underline{v}$