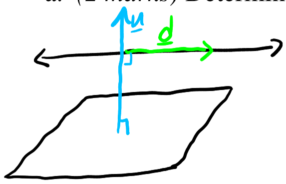
Dawson College: **Linear Algebra**: 201-105-DW-S5: Fall 2022: **Quiz 10** name: Y. Lamontagne

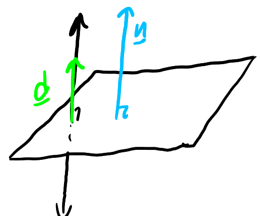
Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Given the plane $x+y+z=0$ and the line $(x,y,z)=(1+t,2+2t,3+3t)$ where $t\in\mathbb{R}$.

a. *(2 marks)* Determine whether the line is perpendicular to the plane, parallel or neither. Justify.

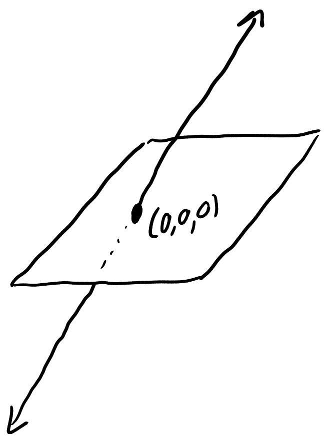


The line and the plane are parallel iff $\underline{n} \perp \underline{d}$.
$\underline{n}\cdot\underline{d} = (1,1,1)\cdot(1,2,3) = 6 \neq 0$ $\therefore$ not parallel.



The line and the plane are orthogonal iff $\underline{n} \parallel \underline{d}$.
Since $\underline{n}$ and $\underline{d}$ are not multiples of each other, the line and the plane are not perpendicular.

b. *(3 marks)* Find the point of intersection between the line and the plane if it exists.



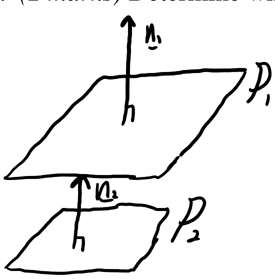
$$(1+t)+(2+2t)+(3+3t)=0$$
$$6+6t=0$$
$$t=-1$$

$$(x,y,z)=(1+(-1),\ 2+2(-1),\ 3+3(-1))=(0,0,0)$$

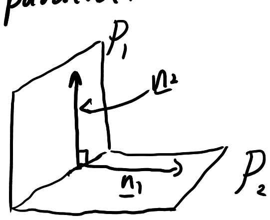
$\therefore$ point of intersection is $(0,0,0)$.

Questions 2. Given the planes $2x-3z=7$ and $y+2z=4$.

a. *(2 marks)* Determine whether the two planes are perpendicular to each other, parallel or neither. Justify.



P_1 and P_2 are parallel if $\underline{n}_1 \parallel \underline{n}_2$. Since $\nexists k$ s.t. $\underline{n}_1 = k\underline{n}_2$ the planes are not parallel.



P_1 and P_2 are perpendicular if $n_1 \perp n_2$. Since $\underline{n}_1\cdot\underline{n}_2 = 2(0)+(0)(1)+(-3)(2) \neq 0$

$\therefore$ P_1 and P_2 are not perpendicular.

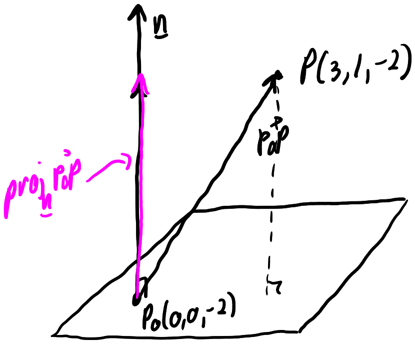
b. *(3 marks)* Find the intersection between the planes if it exists. Since the planes are not parallel, they intersect.

$$\begin{bmatrix} 2 & 0 & -3 & 7 \\ 0 & 1 & 2 & 4 \end{bmatrix} \sim \frac{1}{2}R_1 \to R_1 \begin{bmatrix} 1 & 0 & -3/2 & 7/2 \\ 0 & 1 & 2 & 4 \end{bmatrix}$$

Let $z=t$ $\quad x = \frac{7}{2}+\frac{3}{2}t$
$\quad y = 4-2t$

$\therefore (x,y,z) = (\frac{7}{2}+\frac{3}{3}t,\ 4-2t,\ t)$ $\quad t\in\mathbb{R}$

Questions 3. *(4 marks)* Using projection(s) find the shortest distance between $P(3,1,-2)$ and $x+2y-2z=4$.



Let $x=y=0 \Rightarrow 0+2(0)-2z=4$, $z=-2$ $\therefore P_0(0,0,-2)$ is a point on the plane

$\vec{P_0P} = \underline{P}-\underline{P_0} = (3,1,-2)-(0,0,-2) = (3,1,0)$

$$\begin{aligned}\text{distance} &= \|\text{proj}_{\underline{n}}\vec{P_0P}\| \\ &= \left\|\tfrac{5}{9}(1,2,-2)\right\| \\ &= \tfrac{5}{9}\|(1,2,-2)\| \\ &= \tfrac{5}{9}\sqrt{1^2+2^2+(-2)^2} \\ &= \tfrac{5}{9}\sqrt{9}\end{aligned}$$

$$\begin{aligned}\text{proj}_{\underline{n}}\vec{P_0P} &= \frac{\underline{n}\cdot\vec{P_0P}}{\underline{n}\cdot\underline{n}}\underline{n} \\ &= \frac{(1,2,-2)\cdot(3,1,0)}{(1,2,-2)\cdot(1,2,-2)}(1,2,-2) \\ &= \frac{5}{1+4+4}(1,2,-2) \\ &= \frac{5}{9}(1,2,-2)\end{aligned}$$

Question Bonus. *(2 marks)* A former Prime Minister of Canada defined a proof as

> I don't know — a proof is a proof. What kind of a proof? It's a proof. A proof is a proof, and when you have a good proof, it's because it's proven.

In your own words correctly define proof.