**Dawson College**: **Linear Algebra**: 201-105-DW-S5: Fall 2022: **Quiz 12** name: _____

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.**[1] The Silly Nut Company makes two mixtures of nuts: Mixture A and Mixture B. A pound of Mixture A contains 12 oz of peanuts, 3 oz of almonds and 1 oz of cashews and sells for \$4. A pound of Mixture B contains 12 oz of peanuts, 2 oz of almonds and 2 oz of cashews and sells for \$5. The company has 1080 lb. of peanuts, 240 lb. of almonds, 160 lb. of cashews. How many pounds of each of mixtures A and B should the company make to maximize profit? (Hint: Use consistent units. Work the entire problem in pounds by converting all values given in ounces into fractions of pounds, 1 lb = 16 oz).

a. (1 mark) Determine the objective function.

$Z = 4A + 5B$

b. (3 marks) Find the constraints that define the feasible region.

$\frac{12}{16}A + \frac{12}{16}B \leq 1080$

$\frac{3}{16}A + \frac{2}{16}B \leq 240$

$\frac{1}{16}A + \frac{2}{16}B \leq 160$

**Questions 2.**[2] *(4 marks)* Consider the objective function $Z = 4x + 3y$ subject to the following constraints:

$$\begin{cases} x+y \geq 1 \\ x+2y \leq 4 \\ 2x+y \leq 4 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

If the vertices (corners) of the feasible region are $(1,0), (2,0), (0,2), (0,1), (4/3, 4/3)$. Find the maximum and minimum values of $Z$ and at what $(x,y)$ point they are reached. And justify!

| x | y | $Z = 4x+3y$ |
|---|---|---|
| 1 | 0 | 4 |
| 2 | 0 | 8 |
| 0 | 2 | 6 |
| 0 | 1 | 3 |
| 4/3 | 4/3 | $4(\frac{4}{3}) + 3(\frac{4}{3}) = \frac{16}{3} + 4 = \frac{16}{3} + \frac{12}{3} = \frac{28}{3} \approx 9.3$ |

Minimum is 3 when $x=0$ and $y=1$

Maximum is $\frac{28}{3}$ when $x = y = \frac{4}{3}$

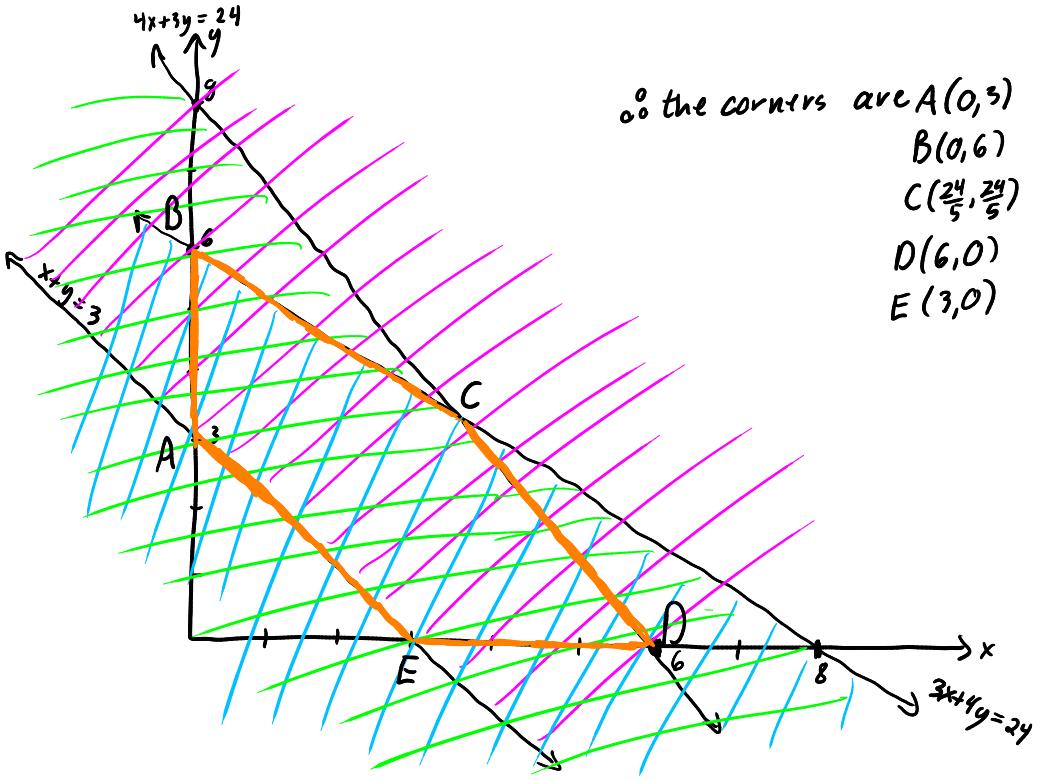
The above conclusion is true since linear objective function are optimized at the vertices (corners) of bounded feasible regions.

[1] From https://math.libretexts.org/Bookshelves/Applied_Mathematics/Applied_Finite_Mathematics_(Sekhon_and_Bloom)

[2] modified from WeBWorK

**Questions 3.**[3] Sketch the feasible region given by the inequalities below. And also find the feasible region vertices (coners). Show your work!

$$\begin{cases} x+y \geq 3 \\ 3x+4y \leq 24 \\ 4x+3y \leq 24 \\ x \geq 0 \\ y \geq 0 \end{cases}$$




∴ the corners are $A(0,3)$, $B(0,6)$, $C(\frac{24}{5},\frac{24}{5})$, $D(6,0)$, $E(3,0)$

To find C, lets determine the intersection between $3x+4y=24$ and $4x+3y=24$

$$\begin{bmatrix} 3 & 4 & 24 \\ 4 & 3 & 24 \end{bmatrix} \sim {}_{3R_2 \to R_2} \begin{bmatrix} 3 & 4 & 24 \\ 12 & 9 & 72 \end{bmatrix}$$

$$\sim {}_{-4R_1+R_2 \to R_2} \begin{bmatrix} 3 & 4 & 24 \\ 0 & -7 & -24 \end{bmatrix}$$

$$\sim {}_{7R_1 \to R_1} \begin{bmatrix} 21 & 28 & 168 \\ 0 & -7 & -24 \end{bmatrix}$$

$$\sim {}_{4R_2+R_1 \to R_1} \begin{bmatrix} 21 & 0 & 72 \\ 0 & -7 & -24 \end{bmatrix}$$

$$\sim {}_{\substack{\frac{1}{21}R_1 \to R_1 \\ -\frac{1}{7}R_2 \to R_2}} \begin{bmatrix} 1 & 0 & 24/7 \\ 0 & 1 & 24/7 \end{bmatrix}$$

∴ $C(\frac{24}{7},\frac{24}{7})$

[3] modified from WeBWorK