Dawson College: **Linear Algebra**: 201-105-DW-S5: Fall 2022: **Quiz 13** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(10 marks)* Use the simplex method to solve the linear programming problem: Maximize the objective function: $Z = 5x_1 + 2x_2 + 8x_3$ subject to

$$\begin{cases} 2x_1 - 4x_2 + x_3 \leq 42 \\ 2x_1 + 3x_2 - x_3 \leq 42 \\ 6x_1 - x_2 + 3x_3 \leq 42 \end{cases}.$$

$2x_1 - 4x_2 + x_3 + s_1 = 42$
$2x_1 + 3x_2 - x_3 + s_2 = 42$
$6x_1 - x_2 + 3x_3 + s_3 = 42$
$-5x_1 - 2x_2 - 8x_3 + Z = 0$

Explicitly write the final value of the objective function, variables and slack variables.

	x1	x2	x3	s1	s2	s3	Z	
	2	-4	1	1	0	0	0	42
	2	3	-1	0	1	0	0	42
	6	-1	3	0	0	1	0	42
	-5	-2	-8	0	0	0	1	0

$r_1 = 42/1 = 42$
$r_3 = 42/3 = 14$ ← pivot row
↑ pivot col

	x1	x2	x3	s1	s2	s3	Z	
	2	-4	1	1	0	0	0	42
	2	3	-1	0	1	0	0	42
	2	-1/3	1	0	0	1/3	0	14
$\frac{1}{3}R_3 \to R_3$	-5	-2	-8	0	0	0	1	0

	x1	x2	x3	s1	s2	s3	Z	
	0	-11/3	0	1	0	-1/3	0	28
$-R_3+R_1 \to R_1$	4	8/3	0	0	1	1/3	0	56
$R_3+R_2 \to R_2$	2	-1/3	1	0	0	1/3	0	14
$8R_3+R_4 \to R_4$	11	-14/3	0	0	0	8/3	1	112

← pivot row
↑ pivot col

	x1	x2	x3	s1	s2	s3	Z	
	0	-11/3	0	1	0	-1/3	0	28
	3/2	1	0	0	3/8	1/8	0	21
	2	-1/3	1	0	0	1/3	0	14
$\frac{3}{8}R_2 \to R_2$	11	-14/3	0	0	0	8/3	1	112

	x1	x2	x3	s1	s2	s3	Z	
	11/2	0	0	1	11/8	1/8	0	105
	3/2	1	0	0	3/8	1/8	0	21
$11/3R_2+R_1 \to R_1$	5/2	0	1	0	1/8	3/8	0	21
$\frac{1}{3}R_2+R_3 \to R_3$	18	0	0	0	7/4	13/4	1	210
$\frac{14}{3}R_2+R_4 \to R_4$								

a) $X_1 = 0$
$X_2 = 21$
$X_3 = 21$
$S_1 = 105$
$S_2 = 0$
$S_3 = 0$
$Z = 210$

Bonus Question. *(2 marks)* The barber is the "one who shaves all those, and those only, who do not shave themselves". The question is, does the barber shave himself?