Dawson College: **Linear Algebra**: 201-105-DW-S5: Fall 2022: **Quiz 2** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* The augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system.

$$\begin{matrix} x_1 & x_2 & x_3 & x_4 & b \end{matrix}$$
$$\begin{bmatrix} 1 & 2 & 0 & -5 & 6 \\ 0 & 0 & 1 & -9 & 3 \end{bmatrix}$$

Let $x_2 = s$, $x_4 = t$, $s,t \in \mathbb{R}$

and it follows that $x_1 = 6 - 2s + 5t$, $x_3 = 3 + 9t$

$\therefore \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 6-2s+5t \\ s \\ 3+9t \\ t \end{pmatrix}$ $s,t \in \mathbb{R}$

Question 2. *(2 marks)* The augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system.

$$\begin{matrix} x_1 & x_2 & x_3 & b \end{matrix}$$
$$\begin{bmatrix} 1 & -101 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The system has no solutions since $\nexists x_1, x_2, x_3$ that satisfy $0x_1 + 0x_2 + 0x_3 = 1$

Question 3. *(5 marks)* Determine the values of p for which the system has no solutions, exactly one solution, or infinitely many solutions

$$\begin{array}{rcrcrcl} -x & + & 4y & - & 2z & = & 1 \\ -2x & + & 10y & + & (2p-4)z & = & 6 \\ 3x & - & 11y & - & (p^2+6)z & = & 5p-1 \end{array}$$

$$\begin{bmatrix} -1 & 4 & -2 & 1 \\ -2 & 10 & (2p-4) & 6 \\ 3 & -11 & (p^2+6) & 5p-1 \end{bmatrix}$$

$$\sim \begin{matrix} -2R_1 + R_2 \to R_2 \\ 3R_1 + R_3 \to R_3 \end{matrix} \begin{bmatrix} -1 & 4 & -2 & 1 \\ 0 & 2 & 2p & 4 \\ 0 & 1 & p^2 & 5p-2 \end{bmatrix}$$

$$\sim \tfrac{1}{2}R_2 \to R_2 \begin{bmatrix} -1 & 4 & -2 & 1 \\ 0 & 1 & p & 2 \\ 0 & 1 & p^2 & 5p-2 \end{bmatrix}$$

$$\sim -R_2 + R_3 \to R_3 \begin{bmatrix} -1 & 4 & -2 & 1 \\ 0 & 1 & p & 2 \\ 0 & 0 & p^2-p & 5p \end{bmatrix}$$

unique solution: # leading entries in var. col. = # var

$p^2 - p \neq 0$

$p(p-1) \neq 0$

$p \neq 0$ and $p-1 \neq 0$, $p \neq 1$

$\therefore p \neq 0, 1$

no solution: leading entry in constant col.

$p^2 - p = 0$ and $5p \neq 0$

$p(p-1) = 0$, $p \neq 0$

$p = 0$ or $p = 1$

$\therefore p = 1$

infinitely many solutions: # leading entry < # var

$p^2 - p = 0$ and $5p = 0$

$p(p-1) = 0$, $p = 0$

$p = 0$ or $p = 1$

$\therefore p = 0$