

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.**Question 1.** (5 marks) Find the matrix M if

$$(5M + 4I)^{-1} = \begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix}^T$$

$$(5M + 4I)^{-1} = \begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix}$$

$$\left((5M + 4I)^{-1}\right)^{-1} = \begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix}^{-1}$$

$$5M + 4I = \frac{1}{7(1) - 4(2)} \begin{bmatrix} 1 & -4 \\ -2 & 7 \end{bmatrix}$$

$$5M + 4I = - \begin{bmatrix} 1 & -4 \\ -2 & 7 \end{bmatrix}$$

$$5M = \begin{bmatrix} -1 & 4 \\ 2 & -7 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$5M = \begin{bmatrix} -5 & 4 \\ 2 & -11 \end{bmatrix}$$

$$M = \begin{bmatrix} -1 & 4/5 \\ 2/5 & -11/5 \end{bmatrix}$$

Question 2. (4 marks) Suppose that A, B and C are invertible matrices. Solve for X and simplify as much as possible, where $(2AC^T)^{-1}(CA^T)^T BX = B^{-1}D^T$.

$$\underbrace{((2AC^T)^{-1}(CA^T)^T B)^{-1}}_I (2AC^T)^{-1}(CA^T)^T BX = (2AC^T)^{-1}(CA^T)^T B^{-1}D^T$$

$$X = B^{-1}((CA^T)^T)^{-1}((2AC^T)^{-1})^{-1}B^{-1}D^T$$

$$X = B^{-1}((A^T)^T C^T)^{-1} 2AC^T B^{-1}D^T$$

$$X = B^{-1}(C^T)^{-1}A^{-1}2AC^T B^{-1}D^T$$

$$X = 2B^{-1}(C^T)^{-1}A^{-1}AC^T B^{-1}D^T$$

$$X = 2B^{-1}(C^T)^{-1}I C^T B^{-1}D^T$$

$$X = 2B^{-1}(C^T)^{-1}C^T B^{-1}D^T$$

$$X = 2B^{-1}I B^{-1}D^T$$

$$X = 2B^{-2}D^T$$

Question 3. (3 marks) Show that if a square matrix A satisfies the equation $A^2 + 2A + I = 0$, then A must be invertible. What is the inverse?

We have $A^2 + 2A + I = 0$

$$I = -A^2 - 2A$$

which implies that $I = A(-A - 2I)$ and $I = (-A - 2I)A$

∴ A is invertible and $A^{-1} = -A - 2I$