Dawson College: **Linear Algebra**: 201-105-DW-S5: Fall 2022: **Quiz 4** name: ____________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(5 marks)* Consider the following Gauss-Jordan reduction:

$$\underbrace{\begin{bmatrix} -2 & 5 & 4 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{A} \overset{-4R_3+R_1\to R_1}{\sim} \underbrace{\begin{bmatrix} -2 & 5 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_1A} \overset{R_1\leftrightarrow R_2}{\sim} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_2E_1A} \overset{2R_1+R_2\to R_2}{\sim} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_3E_2E_1A} \overset{\frac{1}{5}R_2\to R_2}{\sim} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_4E_3E_2E_1A} = I$$

Find E_1, E_2, E_3, E_4 and express A as a product of elementary matrices.

E_1: $I_3 \sim -4R_3+R_1\to R_1 \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_1$

E_2: $I_3 \sim R_1\leftrightarrow R_2 \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_2$

E_3: $I_3 \sim 2R_1+R_2\to R_2 \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_3$

E_4: $I_3 \sim \frac{1}{5}R_2\to R_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_4$

$E_4E_3E_2E_1A = I$

A is invertible since its RREF is I

$E_4E_3E_2E_1AA^{-1} = IA^{-1}$

$E_4E_3E_2E_1 = A^{-1}$

$(E_4E_3E_2E_1)^{-1} = (A^{-1})^{-1}$

$A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}$

E_1^{-1}: $I_3 \sim 4R_3+R_1\to R_1 \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_1^{-1}$

E_2^{-1}: $I_3 \sim R_1\leftrightarrow R_2$ E_2

E_3^{-1}: $I_3 \sim -2R_1+R_2\to R_2 \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_3^{-1}$

E_4^{-1}: $I_3 \sim 5R_2\to R_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_4^{-1}$

Question 2. *(5 marks)* Find the inverse of A, if possible.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \begin{matrix} -2R_1+R_2\to R_2 \\ -R_1+R_3\to R_3 \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

$$\sim 2R_2+R_3\to R_3 \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right]$$

$$\sim \begin{matrix} 3R_3+R_1\to R_1 \\ -3R_3+R_2\to R_2 \\ -R_3\to R_3 \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & 6 & 3 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$$\sim -2R_2+R_1\to R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$\therefore$ A is invertible since its RREF is I. And $A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$

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[1] from WeBWorK