Dawson College: **Linear Algebra**: 201-105-DW-S5: Fall 2022: **Quiz 6** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Determine conditions on the b_i's, if any, in order to guarantee that the linear system is consistent. And discuss your result using the Equivalence Theorem.

$$\begin{cases} x_1 - 2x_2 - x_3 = b_1 \\ -4x_1 + 5x_2 + 2x_3 = b_2 \\ -4x_1 + 7x_2 + 4x_3 = b_3 \end{cases}$$

Since the RREF of the coefficient matrix is I (if we had continued with Gauss Jordan), it follows by the equivalence theorem that the system is consistent for all constants.

$$\begin{bmatrix} 1 & -2 & -1 & b_1 \\ -4 & 5 & 2 & b_2 \\ -4 & 7 & 4 & b_3 \end{bmatrix}$$

$$\sim \begin{matrix} 4R_1+R_2 \to R_2 \\ 4R_1+R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & -3 & -2 & b_2+4b_1 \\ 0 & -1 & 0 & b_3+4b_1 \end{bmatrix}$$

$$\sim R_2 \leftrightarrow R_3 \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & -1 & 0 & b_3+4b_1 \\ 0 & -3 & -2 & b_2+4b_1 \end{bmatrix}$$

$$\sim -3R_2+R_3 \to R_3 \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & -1 & 0 & b_3+4b_1 \\ 0 & 0 & -2 & -3b_3+b_2-8b_1 \end{bmatrix}$$

$\therefore$ system is consistent forall $b_i \in \mathbb{R}$.

Quations 2. *(1 mark)* Create a symmetric matrix by substituting appropriate numbers for the x's.

$$\begin{bmatrix} 0 & x & x & x \\ 3 & 0 & x & x \\ 7 & -8 & 0 & x \\ 2 & -3 & 9 & 0 \end{bmatrix}$$

3 7 2 -3 -8 9

Question 3. *(4 marks)* If B and C are $n \times n$ matrices such that $A = B^T C + C^T B$ is invertible then show A^{-1} is symmetric.

premise:

- $A = B^TC + C^TB$ is invertible

conclusion:

- A^{-1} is symmetric

We want to show that $(A^{-1})^T = A^{-1}$

$$\begin{aligned} LHS &= (A^{-1})^T \\ &= (A^T)^{-1} \\ &= ((B^TC + C^TB)^T)^{-1} \\ &= ((B^TC)^T + (C^TB)^T)^{-1} \\ &= (C^T(B^T)^T + B^T(C^T)^T)^{-1} \\ &= (C^TB + B^TC)^{-1} \\ &= (B^TC + C^TB)^{-1} \\ &= A^{-1} \\ &= RHS. \end{aligned}$$