

Books, watches, notes or cell phones are not allowed. The only calculators allowed are the Sharp EL-531**. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. (5 marks) Only using cofactor expansions evaluate

$$\begin{vmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{vmatrix} = \overbrace{a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} + a_{14}C_{14}}^0 \\
 = (-3)(-1)^{3+3} \begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix} + 3(-1)^{4+3} \begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix} \\
 = -3 [a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}] - 3 [a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}] \\
 = -3 \left[3 \begin{vmatrix} 2 & -2 \\ 10 & 2 \end{vmatrix} - 3 \begin{vmatrix} 2 & -2 \\ 2 & 2 \end{vmatrix} + 5 \begin{vmatrix} 2 & 2 \\ 2 & 10 \end{vmatrix} \right] - 3 \left[4(-1)^{3+1} \begin{vmatrix} 3 & 5 \\ 2 & -2 \end{vmatrix} + 1(-1)^{1+2} \begin{vmatrix} 3 & 5 \\ 2 & -2 \end{vmatrix} \right] \\
 = -3 [3(24) - 3(8) + 5(16)] - 3 [4(-16) - (-16)] \\
 = -3 [128] - 3 [-48] \\
 = -240$$

Questions 2. (5 mark) Evaluate $\underbrace{\begin{vmatrix} a-5g & -a+2d & -3g \\ b-5h & -b+2e & -3h \\ c-5i & -c+2f & -3i \end{vmatrix}}_A$, if we know $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -4$

$$|A| = |A^T|$$

$$= \begin{vmatrix} a-5g & b-5h & c-5i \\ -a+2d & -b+2e & -c+2f \\ -3g & -3h & -3i \end{vmatrix}$$

$$= \frac{-1}{3} R_3 \rightarrow R_3 \quad (-3) \begin{vmatrix} a-5g & b-5h & c-5i \\ -a+2d & -b+2e & -c+2f \\ g & h & i \end{vmatrix}$$

$$= 5R_3 + R_1 \rightarrow R_1 \quad -3 \begin{vmatrix} a & b & c \\ -a+2d & -b+2e & -c+2f \\ g & h & i \end{vmatrix}$$

$$= R_1 + R_2 \rightarrow R_2 \quad -3 \begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g & h & i \end{vmatrix}$$

$$= \frac{1}{2} R_2 \rightarrow R_2 \quad (-3)(2) \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (-3)(2)(-4) = 24$$