Dawson College: **Linear Algebra**: 201-105-DW-S5: Fall 2022: **Quiz 8** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Let A and B be 2×2 matrices, where $\det(A)=3$ and $\det(B)=5$. Find $\det(B^TA^{-3}\text{adj}(A)(2AB)^2)$.

$$= \det(B^T)\det(A^{-3})\det(\text{adj}(A))\det((2AB)^2)$$
$$= \det(B)(\det A)^{-3}(\det(A))^{2-1}(\det(2AB))^2$$
$$= 5\cdot \frac{1}{3^{\cancel{3}2}}\cancel{3}\,(2^2\det(AB))^2$$
$$= \frac{5}{9}\,2^4\det(A)\det(B)$$
$$= \frac{5\cdot 16}{\cancel{9}3}(\cancel{3})(5)$$
$$= \frac{5\cdot 16}{3}\cdot 5 = \frac{5^2\cdot 16}{3}$$

Questions 2. Given the linear system $\begin{cases} -2x_1+3x_2+2x_3=1 \\ x_1-x_2+4x_3=2 \\ -3x_1+2x_2+x_3=5 \end{cases}$

a. *(3 marks)* Find the first column of the adjoint of the coefficient matrix of the above system.

$$\text{adj}\,A = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} = \begin{bmatrix} -9 & C_{21} & C_{31} \\ -13 & C_{22} & C_{32} \\ -1 & C_{23} & C_{33} \end{bmatrix}$$

$$C_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix} = -9$$
$$C_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 4 \\ -3 & 1 \end{vmatrix} = -(1+12) = -13$$
$$C_{13} = (-1)^{1+3}\begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix} = 2-3 = -1$$

b. *(4 marks)* Find x_2 of the above system only by using Cramer's Rule.

$$|A| = \begin{vmatrix} -2 & 3 & 2 \\ 1 & -1 & 4 \\ -3 & 2 & 1 \end{vmatrix} = a_{11}C_{11}+a_{12}C_{12}+a_{13}C_{13} = -2\begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & 4 \\ -3 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix} = -2(-9)-3(13)+2(-1) = -23$$

$$|A_2| = \begin{vmatrix} -2 & 1 & 2 \\ 1 & 2 & 4 \\ -3 & 5 & 1 \end{vmatrix} = -2\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 4 \\ -3 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ -3 & 5 \end{vmatrix} = -2(2-20)-(1+12)+2(5+6)$$
$$= -2(-18)-13+2(11)$$
$$= 45$$

$$\therefore\ x_2 = \frac{|A_2|}{|A|} = \frac{45}{-23}$$