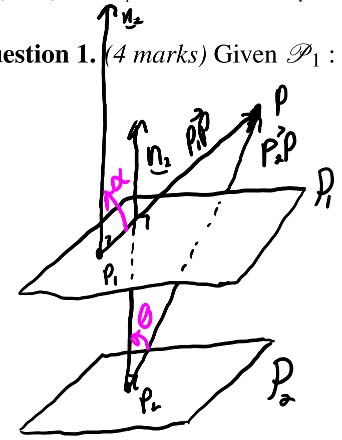
Dawson College: **Linear Algebra** (Science): 201-NYC-05-S7: Fall 2022: **Quiz 10** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Given $\mathscr{P}_1 : 6x-3y+6z=-3$ and $\mathscr{P}_2 : 4x-2y+4z=2$. Determine whether $P(1,2,1)$ is between $\mathscr{P}_1$ and $\mathscr{P}_2$.



$\mathcal{P}_1 \parallel \mathcal{P}_2$ since $\underline{n}_1 \parallel \underline{n}_2$ because $\underline{n}_1 = -2\underline{n}_2$

Let P_1 be a point on $\mathcal{P}_1$, Let $x=z=0 \Rightarrow 6(0)-3y+6(0)=-3$
$y=1$
$\therefore P_1(0,1,0)$

Let P_2 be a point on $\mathcal{P}_2$, Let $x=z=0 \Rightarrow 4(0)-2y+4(0)=2$
$y=-1$
$\therefore P_2(0,-1,0)$

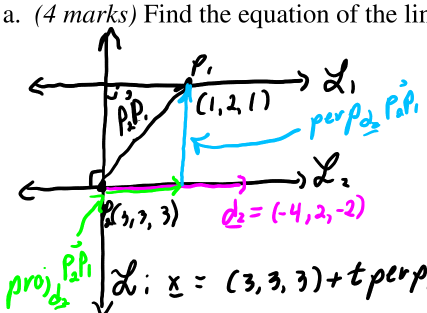
$\vec{P_1P} = \underline{P} - \underline{P}_1 = (1,2,1)-(0,1,0) = (1,1,1)$
$\vec{P_2P} = \underline{P} - \underline{P}_2 = (1,2,1)-(0,-1,0) = (1,3,1)$

$\vec{P_1P} \cdot \underline{n}_2 = (1,1,1)\cdot(4,-2,4) = 6 > 0 \quad \therefore \alpha$ is acute
$\vec{P_2P} \cdot \underline{n}_2 = (1,3,1)\cdot(4,-2,4) = 2 > 0 \quad \therefore \theta$ is acute

$\therefore P$ does not lie between $\mathcal{P}_1$ and $\mathcal{P}_2$.

Question 2. Given $\mathscr{L}_1 : \mathbf{x} = (1,2,1)+t(2,-1,1),\ t\in\mathbb{R}$ and $\mathscr{L}_2 : \mathbf{x} = (3,3,3)+t(-4,2,-2),\ t\in\mathbb{R}$

a. *(4 marks)* Find the equation of the line that passes throught the point $(3,3,3)$ and intersect perpendicularly $\mathscr{L}_1$.



$\mathcal{L}: \underline{x} = (3,3,3) + t\, \text{perp}_{\underline{d}_2} \vec{P_2P_1}$
$= (3,3,3) + t\left(\frac{-2}{6}, \frac{-11}{6}, \frac{-7}{6}\right)$

$\vec{P_2P_1} = \underline{P}_1 - \underline{P}_2 = (1,2,1)-(3,3,3) = (-2,-1,-2)$

$\text{proj}_{\underline{d}_2} \vec{P_2P_1} = \frac{\underline{d}_2 \cdot \vec{P_2P_1}}{\underline{d}_2 \cdot \underline{d}_2} \underline{d}_2$

$= \frac{(-4,2,-2)\cdot(-2,-1,-2)}{(-4,2,-2)\cdot(-4,2,-2)}(-4,2,-2)$

$= \frac{8-2+4}{16+4+4}(-4,2,-2) = \frac{10}{24}(-4,2,-2)$

$= \frac{5}{12}(-4,2,-2)$

$= \frac{5}{6}(-2,1,-1)$

note: $\mathcal{L}_1 \parallel \mathcal{L}_2$ since $\underline{d}_1 \parallel \underline{d}_2$ because $\underline{d}_2 = -2\underline{d}_1$

$\text{perp}_{\underline{d}_2} \vec{P_2P_1} = \vec{P_2P_1} - \text{proj}_{\underline{d}_2} \vec{P_2P_1}$

$= (-2,-1,-2) - \frac{5}{6}(-2,1,-1)$

$= \left(\frac{-2}{6}, \frac{-11}{6}, \frac{-7}{6}\right)$

b. *(2 marks)* Find the parametric equation of the plane that contains $\mathscr{L}_1$ and $\mathscr{L}_2$.

$\mathcal{P}: \underline{x} = \underline{P}_0 + s\underline{d}_1 + t\underline{d}_2 \quad s,t \in \mathbb{R}$
$= (3,3,3) + s(2,-1,1) + t\, \text{perp}_{\underline{d}_2} \vec{P_2P_1}$

Question 3. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

1. *(3 marks)* The general solution of the nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$ can be obtained by adding $\mathbf{b}$ to the general solution of the homogeneous linear system $A\mathbf{x} = \mathbf{0}$.

False

$\underset{A}{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}} \underset{\underline{x}}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{\underline{b}}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$ has a unique solution $(0,1)$ but the general solution of $A\underline{x}=\underline{0}$ is the trivial solution since $\det(A) \neq 0$.

And $(0,0) + \underline{b} = (0,0) + (1,1)$ is not the solution set of $A\underline{x} = \underline{b}$.

Question Bonus. *(2 marks)* A former Prime Minister of Canada defined a proof as

> I don't know — a proof is a proof. What kind of a proof? It's a proof. A proof is a proof, and when you have a good proof, it's because it's proven.

In your own words correctly define proof.