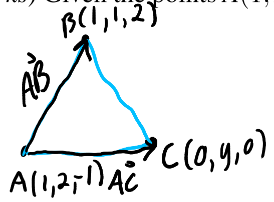
Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S7: Fall 2022: **Quiz 11** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Given the points $A(1,2,-1)$ and $B(1,1,2)$. Find the point C on the y-axis such that the area of the triangle ABC is $\sqrt{10}/2$.



$\vec{AB} = \underline{B} - \underline{A} = (1,1,2) - (1,2,-1)$
$= (0,-1,3)$

$\vec{AC} = \underline{C} - \underline{A} = (0,y,0) - (1,2,-1)$
$= (-1, y-2, 1)$

$$\begin{array}{cc} 0 & -1 \\ -1 & y-2 \\ 3 & 1 \end{array} \qquad \vec{AB} \times \vec{AC} = \left(\begin{vmatrix} -1 & y-2 \\ 3 & 1 \end{vmatrix}, -\begin{vmatrix} 0 & -1 \\ 3 & 1 \end{vmatrix}, \begin{vmatrix} 0 & -1 \\ -1 & y-2 \end{vmatrix} \right)$$
$$= (-1-3(y-2), -3, -1)$$
$$= (5-3y, -3, -1)$$

$\text{Area} = \dfrac{\|\vec{AB} \times \vec{AC}\|}{2}$

$\dfrac{\sqrt{10}}{2} = \dfrac{\|\vec{AB} \times \vec{AC}\|}{2}$

$\sqrt{10} = \|\vec{AB} \times \vec{AC}\|$

$\sqrt{10} = \|(5-3y, -3, -1)\|$

$\sqrt{10} = \sqrt{(5-3y)^2 + (-3)^2 + (-1)^2}$

$\sqrt{10} = \sqrt{(5-3y)^2 + 10}$

$10 = (5-3y)^2 + 10$

$0 = (5-3y)^2$

$0 = 5-3y$

$3y = 5$

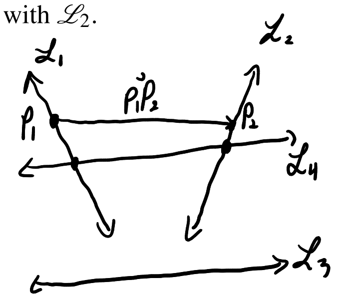
$y = \frac{5}{3}$

$\therefore C(0, \frac{5}{3}, 0)$

Question 2. *(5 marks)* Given the following lines which are all skew to each other:

$$\begin{array}{lcllcl} \mathscr{L}_1 & : & (x,y,z) = (1,0,0) & + & t_1(1,2,0) \\ \mathscr{L}_2 & : & (x,y,z) = (1,1,0) & + & t_2(1,0,1) \\ \mathscr{L}_3 & : & (x,y,z) = (1,0,1) & + & t_3(1,2,3) \end{array}$$

here t_1, t_2, $t_3 \in \mathbb{R}$. Consider a line $\mathscr{L}_4$ that is parallel to $\mathscr{L}_3$ and intersects both $\mathscr{L}_1$ and $\mathscr{L}_2$. Find the points of intersection of $\mathscr{L}_4$ with $\mathscr{L}_1$ and $\mathscr{L}_4$ with $\mathscr{L}_2$.



$\vec{P_1P_2} = (1,1,0) + t_2(1,0,1) - [(1,0,0) + t_1(1,2,0)] = (t_2 - t_1, 1 - 2t_1, t_2)$

So $k\underline{d_3} = \vec{P_1P_2}$

$k(1,2,3) = (t_2 - t_1, 1-2t_1, t_2)$

$$\begin{aligned} -t_1 + t_2 - k &= 0 \\ -2t_1 \quad\;\; - 2k &= -1 \\ t_2 - 3k &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ -2 & 0 & -2 & -1 \\ 0 & 1 & -3 & 0 \end{array}\right]$$

$$\underset{-2R_1+R_2 \to R_2}{\sim} \left[\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & -2 & 0 & -1 \\ 0 & 1 & -3 & 0 \end{array}\right]$$

$$\underset{\substack{\frac{1}{2}R_2+R_1 \to R_1 \\ -\frac{1}{2}R_2 \to R_2 \\ \frac{1}{2}R_2+R_3 \to R_3}}{\sim} \left[\begin{array}{ccc|c} -1 & 0 & -1 & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -3 & -\frac{1}{2} \end{array}\right]$$

$$\underset{\substack{-R_1 \to R_1 \\ -\frac{1}{3}R_3 \to R_3}}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{6} \end{array}\right]$$

$$\underset{-R_3+R_1 \to R_1}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{6} \end{array}\right]$$

$t_1 = \frac{1}{3}$
$t_2 = \frac{1}{2}$

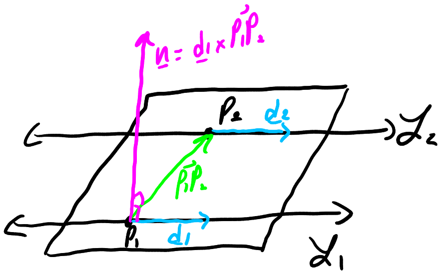
$\therefore$ point of intersection between $\mathscr{L}_1$ and $\mathscr{L}_4$.

$(x,y,z) = (1,0,0) + \frac{1}{3}(1,2,0)$
$= (\frac{4}{3}, \frac{2}{3}, 0)$

between $\mathscr{L}_2$ and $\mathscr{L}_4$

$(x,y,z) = (1,1,0) + \frac{1}{2}(1,0,1)$
$= (\frac{3}{2}, 1, \frac{1}{2})$

Question 3. *(3 marks)* Given $\mathscr{L}_1 : \mathbf{x} = (1,2,1) + t(2,-1,1)$, $t \in \mathbb{R}$ and $\mathscr{L}_2 : \mathbf{x} = (3,3,3) + t(-4,2,-2)$, $t \in \mathbb{R}$. Find the general equation of the plane that contains $\mathscr{L}_1$ and $\mathscr{L}_2$.



$\underline{d}_1 = (2,-1,1)$, $\vec{P_1P_2} = \underline{P}_2 - \underline{P}_1 = (3,3,3) - (1,2,1) = (2,1,2)$

$\underline{n} = \underline{d}_1 \times \vec{P_1P_2} = \left(\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix}, - \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix}, \begin{vmatrix} 2 & 2 \\ -1 & 1 \end{vmatrix} \right)$

$\begin{matrix} 2 & 2 \\ -1 & 1 \\ 1 & 2 \end{matrix}$

$= (-3, -2, 4)$

$-3x - 2y + 4z = d$

sub P_1

$-3(1) - 2(2) + 4(1) = d$

$-3 = d$

$\therefore -3x - 2y + 4z = -3$

Question 4. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

(3 marks) If $\vec{u}, \vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^3$, where $\vec{u} \neq \vec{0}$ and $\vec{u} \times \vec{v} = \vec{u} \times \vec{w}$, then $\vec{v} = \vec{w}$.

False,

Let $\underline{u} = (1,0,0)$, $\underline{v} = 2\underline{u}$, $\underline{w} = 3\underline{u}$, then $\underline{u} \times \underline{v} = \underline{u} \times (2\underline{u}) = 2(\underline{u} \times \underline{u}) = 2\underline{0} = \underline{0}$

$\underline{u} \times \underline{w} = \underline{u} \times (3\underline{u}) = 3(\underline{u} \times \underline{u}) = 3\underline{0} = \underline{0}$

but $\underline{v} \neq \underline{w}$

Question Bonus. *(2 marks)* Discuss and give a correct analysis of the following

Let $R\{x \mid x \notin x\}$, then $R \in R \leftrightarrow R \notin R$