Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S7: Fall 2022: **Quiz 12** name: ______________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Consider the set

$$V = \{(x,\, y) \mid x + y = 2\}$$

under the following operations:

$$(x_1,\, y_1) + (x_2,\, y_2) = (x_1 \cdot x_2,\, y_1 + y_2 - 1) \qquad k \cdot (x,\, y) = (kx - k + 1,\, y^k)$$

a. *(1 mark)* Find $2 \cdot (3, -1)$.

b. *(3 marks)* Show that V contains a zero vector (in the sense of a vector space).

c. *(3 marks)* Does V contain an additive inverse for $\vec{v} = (-1, 3)$? If so, find it. Justify.

d. *(2 marks)* Is V with the given operations a vector space? Justify.

Question 2. *(3 marks)* If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

There does not exist a vector space with two elements.

Bonus Question.
definition:[1] A *group* G is a set of elements satisfying the four conditions below, relative to some binary operation.

1. $\exists e \in G$ s.t. $\forall g \in G : e \cdot g = g \cdot e = g$. (Identity.)
2. $\forall x, y, z \in G :\ (x \cdot y) \cdot z = x \cdot (y \cdot z)$. (Associativity.)
3. $\forall x \in G, \exists y \in G$ s.t. $x \cdot y = y \cdot x = e$. (Inverse.)
4. $\forall x, y \in G : x \cdot y \in G$. (Closure.)

a. *(2 marks)* Show that the identity is unique.

b. *(2 marks)* Is a vector space a group?

[1] https://web.williams.edu/Mathematics/sjmiller/public_html/mathlab/public_html/handouts/GroupTheoryIntro.tex