

Question 1. (5 marks) Prove that if $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent and \vec{v}_3 does not lie in $\text{span}\{\vec{v}_1, \vec{v}_2\}$, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.

Question 2. Consider the subspace $H = \left\{ A \mid A \in \mathcal{M}_{2 \times 2} \text{ and } \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} A = A \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$.

- (1 marks) Is $\mathbf{0}$ (the 2×2 zero matrix) in H ?
- (1 marks) Is I (the 2×2 identity matrix) in H ?
- (1 mark) For what a is $\begin{bmatrix} 2 & 2 \\ 3 & a \end{bmatrix}$ in H ?
- (4 marks) Find a basis for H .
- (2 marks) Express the matrix you found in part c. relative to the basis found in part d., if possible.