Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S7: Fall 2022: **Quiz 14** name: P. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Prove that if $\{\vec{v}1, \vec{v}_2\}$ is linearly independent and $\vec{v}_3$ does not lie in span$\{\vec{v}_1, \vec{v}_2\}$, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.

Suppose $\{\underline{V_1}, \underline{V_2}, \underline{V_3}\}$ is linearly dependent then $\exists c_i \neq 0$ s.t. $c_1\underline{V_1} + c_2\underline{V_2} + c_3\underline{V_3} = \underline{0}$

Case: $C_3 = 0 \Rightarrow C_1 \neq 0$ or $C_2 \neq 0$

$\underline{0} = C_1\underline{V_1} + C_2\underline{V_2} + C_3\underline{V_3}$

$\underline{0} = C_1\underline{V_1} + C_2\underline{V_2} + 0\underline{V_3}$

$\underline{0} = C_1\underline{V_1} + C_2\underline{V_2} \Rightarrow \{\underline{V_1}, \underline{V_2}\}$ is lin. dep ↯

since $\{\underline{V_1}, \underline{V_2}\}$ is lin. ind. by premise.

Case: $C_3 \neq 0 \Rightarrow \underline{0} = C_1\underline{V_1} + C_2\underline{V_2} + C_3\underline{V_3}$

$\underline{V_3} = -\frac{C_1}{C_3}\underline{V_1} - \frac{C_2}{C_3}\underline{V_2}$

$\Rightarrow \underline{V_3} \in$ span $\{\underline{V_1}, \underline{V_2}\}$ ↯ since $\underline{V_3} \notin$ span $\{\underline{V_1}, \underline{V_2}\}$ by premise.

$\therefore \{\underline{V_1}, \underline{V_2}, \underline{V_3}\}$ is lin. ind

Question 2. Consider the subspace $H = \left\{ A \,\middle|\, A \in \mathscr{M}_{2\times 2} \text{ and } \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} A = A \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$.

a. *(1 marks)* Is **0** (the 2×2 zero matrix) in H?

b. *(1 marks)* Is I (the 2×2 identity matrix) in H?

c. *(1 mark)* For what a is $\begin{bmatrix} 2 & 2 \\ 3 & a \end{bmatrix}$ in H?

d. *(4 marks)* Find a basis for H.

e. *(2 marks)* Express the matrix you found in part c. relative to the basis found in part d., if possible.

a) LHS $= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

RHS $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore 0 \in H$

b) LHS $= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

RHS $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \neq$ LHS

$\therefore I \notin H$

c) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 3 & a \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & a \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 2 & 2 \\ -3 & -a \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ a & 3 \end{bmatrix} \Rightarrow a = -3$

d) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in H$

then $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} a & b \\ -c & -d \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$

$\therefore a = b$ and $d = -c$ and $A = \begin{bmatrix} a & a \\ c & -c \end{bmatrix}$

$A = a\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}}_{M_1} + c\underbrace{\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}}_{M_2}$

$\therefore H = $ span $\{M_1, M_2\}$

$\{M_1, M_2\}$ is lin. ind since M_1 and M_2 are not multiples of each other. $\therefore \{M_1, M_2\}$ is a basis of H.

e) $\left(\begin{bmatrix} 2 & 2 \\ 3 & -3 \end{bmatrix}\right)_B = (C_1, C_2) = (2, 3)$

$\begin{bmatrix} 2 & 2 \\ 3 & -3 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + 3\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}$