Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S7: Fall 2022: **Quiz 1** name: ______________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Given the linear system

$$\begin{cases} x_1 + 2x_2 + x_3 + 3x_4 = 4 \\ 2x_1 + 3x_2 + 3x_3 + 4x_4 = 7 \\ -x_1 + x_2 - 4x_3 + 3x_4 = -1 \\ 3x_1 + 5x_2 + 4x_3 + 7x_4 = 11 \end{cases}$$

a. *(4 marks)* Apply Gauss-Jordan elimination on the augmented matrix of the above system.

b. *(2 marks)* Find the solution set of the above system.

c. *(2 marks)* Find a particular solution for which $x_1 = -4$, and $x_2 = 8$.

d. *(3 marks)* If the augmented matrix of the above system is the coefficient matrix of a homogeneous system, find its solution set.

a) $\begin{bmatrix} 1 & 2 & 1 & 3 & 4 \\ 2 & 3 & 3 & 4 & 7 \\ -1 & 1 & -4 & 3 & -1 \\ 3 & 5 & 4 & 7 & 11 \end{bmatrix} \sim \begin{array}{r} \\ -2R_1+R_2 \to R_2 \\ R_1+R_3 \to R_3 \\ -3R_1+R_4 \to R_4 \end{array} \begin{bmatrix} 1 & 2 & 1 & 3 & 4 \\ 0 & -1 & 1 & -2 & -1 \\ 0 & 3 & -3 & 6 & 3 \\ 0 & -1 & 1 & -2 & -1 \end{bmatrix}$

$\sim \begin{array}{r} 2R_2+R_1 \to R_1 \\ -R_2 \to R_2 \\ 3R_2+R_3 \to R_3 \\ -R_2+R_4 \to R_4 \end{array} \begin{bmatrix} 1 & 0 & 3 & -1 & 2 \\ 0 & 1 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

b) Let $x_3 = s$, $x_4 = t$, $s,t \in \mathbb{R}$

$x_1 = 2-3s+t$

$x_2 = 1+s-2t$

$\therefore (x_1,x_2,x_3,x_4) = (2-3s+t,\ 1+s-2t,\ s,\ t)$

c) $\left.\begin{array}{l} -4 = 2-3s+t \\ 8 = 1+s-2t \end{array}\right\} \Rightarrow \begin{array}{l} -6 = -3s+t \\ 7 = s-2t \end{array}$

$\begin{bmatrix} 1 & -2 & 7 \\ -3 & 1 & -6 \end{bmatrix} \sim 3R_1+R_2 \to R_2 \begin{bmatrix} 1 & -2 & 7 \\ 0 & -5 & 15 \end{bmatrix}$

$\sim -\frac{1}{5}R_2 \to R_2 \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -3 \end{bmatrix}$

$\sim 2R_2+R_1 \to R_1 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \end{bmatrix}$

$\therefore s=1,\ t=-3$

$\therefore (x_1,x_2,x_3,x_4) = (2-3(1)-3,\ 1+1-2(-3),\ 1,\ -3)$
$= (-4, 8, 1, -3)$

d) $\begin{bmatrix} 1 & 2 & 1 & 3 & 4 & 0 \\ 2 & 3 & 3 & 4 & 7 & 0 \\ -1 & 1 & -4 & 3 & -1 & 0 \\ 3 & 5 & 4 & 7 & 11 & 0 \end{bmatrix}$

$\sim \dots \sim \begin{bmatrix} 1 & 0 & 3 & -1 & 2 & 0 \\ 0 & 1 & -1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Applying Gauss-Jordan will give the same result as the augmented matrix of a) for the coefficient matrix of the homogeneous augmented matrix. Since the additional constant column of zeros does not effect the process.

Let $x_3 = s$, $x_4 = t$, $x_5 = r$, $s,t,r \in \mathbb{R}$

$\therefore$ $x_1 = -3s+t-2r$

$x_2 = s-2t-r$

$x_3 = s$

$x_4 = t$

$x_5 = r$

$s,t,r \in \mathbb{R}$