Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S7: Fall 2022: **Quiz 3** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Solve for A, if possible.

$$3A^T - A = \begin{bmatrix} 2 & 0 \\ 8 & 6 \end{bmatrix}$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$3\begin{bmatrix} a & b \\ c & d \end{bmatrix}^T - \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 8 & 6 \end{bmatrix}$

$3\begin{bmatrix} a & c \\ b & d \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 8 & 6 \end{bmatrix}$

$\begin{bmatrix} 3a-a & 3c-b \\ 3b-c & 3d-d \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 8 & 6 \end{bmatrix}$

$2a = 2 \Rightarrow a = 1$
$2d = 6 \Rightarrow d = 3$

① $3c - b = 0$
② $3b - c = 8$

① + 3②
$-b + 9b = 24$
$8b = 24 \Rightarrow b = 3$

$\to$ sub into ① $3c - 3 = 0$
$c = 1$

$\therefore A = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}$

Question 2. Determine whether the following statements are true or false for any $n \times n$ matrices A and B. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

1. *(3 marks)* If $A^2 = 0$ then $A = 0$.

False, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \neq 0$ but $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

2. *(3 marks)* A square matrix A is *idempotent* if $A^2 = A$. If A and B are idempotent and are commutative then AB is idempotent.

True,
premises:
$A^2 = A$
$B^2 = B$
$AB = BA$
Conclusion: $(AB)^2 = AB$

LHS $= (AB)^2$
$= ABAB$
$= ABBA$ since A & B commute
$= AB^2A$
$= ABA$ since B is idempotent
$= AAB$ since A and B commute
$= A^2B$
$= AB$ since A is idempotent.
$=$ RHS

3. *(3 marks)* If A is any matrix, then $\mathrm{tr}(A^TA) \geq 0$.

True,
Let $B = A^TA$

$\mathrm{tr}(A^TA) = \mathrm{tr}(B)$
$= b_{11} + b_{22} + \ldots + b_{nn}$
≥ 0

where b_{ii} is the ij-product of the i^{th} row of A^T and i^{th} column of A. But the i^{th} row of A^T are the entries of the i^{th} column of A.

$\therefore b_{ii} = \underbrace{a_{1i}^2}_{\geq 0} + \underbrace{a_{2i}^2}_{\geq 0} + \ldots + \underbrace{a_{ni}^2}_{\geq 0} \geq 0$