Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S7: Fall 2022: **Quiz 4** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(5 marks)* Given that A and B are invertible with $B = B^T$, solve for matrix X, if possible. Your answer should be expressed as a single term.

$$B^T AX - A = (B-I)(B+I)A$$

$$BAX = [B^2 - B + B - I]A + A$$
$$BAX = [B^2 - I]A + A$$
$$BAX = B^2A - A + A$$
$$BAX = B^2A$$
$$(BA)^{-1}(BA)X = (BA)^{-1}B^2A$$
$$IX = A^{-1}B^{-1}BBA$$
$$X = A^{-1}IBA$$
$$X = A^{-1}BA$$

Question 2. Determine whether the following statements are true or false for any $n \times n$ matrices A and B. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

1. *(3 marks)* The sum of two invertible matrices of the same size must be invertible.

False. let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, they are both invertible matrices since $ad - bc \neq 0$

But $A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ which is not invertible.

2. *(3 marks)* A square matrix A is *idempotent* if $A^2 = A$. If A is idempotent then A is singular or $A = I$. *Hint: Its true! And prove it by contradiction.*

premise: $A^2 = A$

conclusion: A is singular or $A = I$

Lets prove by contradiction, suppose A is invertible and $A \neq I$.

$$A^2 = A$$
$$AA = A$$
$$A^{-1}AA = A^{-1}A$$
$$IA = I$$
$$A = I \quad \lightning \text{ since } A \neq I$$

$\therefore$ A is singular or $A = I$

[1] from a past John Abbott final examination