

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531\*\*. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** (4 marks) Let  $A$  be an  $n \times m$  matrix, such that  $A^T A = I_m$ . Show that  $I_n - 2AA^T$  is its own inverse and symmetric.

**Question 2.** (2 marks) If  $(1, 2, 3, 4, 5)$  and  $(4, 0, 4, 3, 1)$  are both solutions of a system of 13 linear equations find a third solution of the system.

**Question 3.** Determine whether the following statements are true or false for any  $n \times n$  matrices  $A$  and  $B$ . If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

1. (3 marks) If  $A$  and  $B$  are square matrices such that  $AB$  can be expressed as a product of elementary matrices, then the system  $A\mathbf{x} = \mathbf{b}$  has exactly one solution.