Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S7: Fall 2022: **Quiz 6** name: ______________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Let A be an $n \times m$ matrix, such that $A^T A = I_m$. Show that $I_n - 2AA^T$ is its own inverse and symmetric.

$I_n - 2AA^T$ is its own inverse iff $(I_n - 2AA^T)(I_n - 2AA^T) = I_n$

$$\begin{aligned}(I_n - 2AA^T)(I_n - 2AA^T) &= I_n I_n - I_n(2AA^T) + -2AA^T I_n + (2AA^T)(2AA^T)\\ &= I_n - 4AA^T + 4AA^TAA^T\\ &= I_n - 4AA^T + 4AI_mA^T\\ &= I_n - 4AA^T + 4AA^T\\ &= I_n\end{aligned}$$

$I_n - 2AA^T$ is symmetric iff $(I_n - 2AA^T)^T = I_n - 2AA^T$

$$\begin{aligned}(I_n - 2AA^T)^T &= I_n^T - (2AA^T)^T\\ &= I_n - 2(AA^T)^T\\ &= I_n - 2(A^T)^T A^T\\ &= I_n - 2AA^T\end{aligned}$$

Question 2. *(2 marks)* If $\underbrace{(1,2,3,4,5)}_{\underline{x_1}}$ and $\underbrace{(4,0,4,3,1)}_{\underline{x_2}}$ are both solutions of a system of 13 linear equations find a third solution of the system.

as shown in class if $\underline{x}_1$ and $\underline{x}_2$ are solutions of a linear system then so is $\underline{x} = \underline{x}_1 + k(\underline{x}_2 - \underline{x}_1)$ $\forall k \in \mathbb{R}$.

$$\begin{aligned}\therefore \underline{x}_3 &= \underline{x}_1 + 2(\underline{x}_2 - \underline{x}_1)\\ &= -\underline{x}_1 + 2\underline{x}_2\\ &= (7,-2,5,2,-3)\end{aligned}$$ is an other solution of the system.

Question 3. Determine whether the following statements are true or false for any $n \times n$ matrices A and B. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

1. *(3 marks)* If A and B are square matrices such that AB can be expressed as a product of elementary matrices, then the system $A\mathbf{x} = \mathbf{b}$ has exactly one solution.

True,

since AB can be written as a product of elementary matrices then AB is invertible by the equivalence thm.

$AB(AB)^{-1} = I$

It follows that A is invertible and its inverse is $B(AB)^{-1}$ since A is a square matrix.

And again by the equivalence thm. $AX = 0$ has only the trivial solution since A is invertible.