

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531\*\*. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** (5 marks) Only using elementary operations show that

$$\begin{vmatrix} 1 & x & x^2 & x^3 \\ a & 1 & x & x^2 \\ p & b & 1 & x \\ q & r & c & 1 \end{vmatrix} = (1-ax)(1-bx)(1-cx)$$

$$\begin{vmatrix} 1 & x & x^2 & x^3 \\ a & 1 & x & x^2 \\ p & b & 1 & x \\ q & r & c & 1 \end{vmatrix} \xrightarrow{-aR_1+R_2 \rightarrow R_2} \begin{vmatrix} 1 & x & x^2 & x^3 \\ 0 & 1-ax & x-ax^2 & x^2-ax^3 \\ p & b & 1 & x \\ q & r & c & 1 \end{vmatrix} \xrightarrow{-pR_1+R_3 \rightarrow R_3} \begin{vmatrix} 1 & x & x^2 & x^3 \\ 0 & 1-ax & x-ax^2 & x^2-ax^3 \\ 0 & b-px & 1-px^2 & x-px^3 \\ q & r & c & 1 \end{vmatrix} \xrightarrow{-qR_1+R_4 \rightarrow R_4} \begin{vmatrix} 1 & x & x^2 & x^3 \\ 0 & 1-ax & x-ax^2 & x^2-ax^3 \\ 0 & b-px & 1-px^2 & x-px^3 \\ 0 & r-qx & c-qx^2 & 1-qx^3 \end{vmatrix}$$

Case 1:  $(1-ax) = 0$  then LHS = 0 since  $R_2$ 's entries are 0 and RHS = 0

Case 2:  $(1-ax) \neq 0$  then

$$\xrightarrow{\frac{1}{1-ax} R_2 \rightarrow R_2} \begin{vmatrix} 1 & x & x^2 & x^3 \\ 0 & 1 & x & x^2 \\ 0 & b-px & 1-px^2 & x-px^3 \\ 0 & r-qx & c-qx^2 & 1-qx^3 \end{vmatrix}$$

$$\xrightarrow{\begin{matrix} -(b-px)R_2+R_3 \rightarrow R_3 \\ -(r-qx)R_2+R_4 \rightarrow R_4 \end{matrix}} \begin{vmatrix} 1 & x & x^2 & x^3 \\ 0 & 1 & x & x^2 \\ 0 & 0 & 1-bx & x-bx^2 \\ 0 & 0 & c-r & 1-rx^2 \end{vmatrix}$$

Case 2a:  $(1-bx) = 0$  then LHS = 0 since  $R_3$ 's entries are 0 and RHS = 0

Case 2b:  $(1-bx) \neq 0$  then

better solution on next page

**Question 2.** (5 marks) By first performing a cofactor expansion along the first column show that

$$\begin{vmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & 1 \\ 1 & 0 & 0 & 0 & \dots & 0 & 1 \end{vmatrix} = 1 + (-1)^{n+1}$$

$$\text{LHS} = a_{11}C_{11} + a_{21}C_{21} + \dots + a_{n+1,1}C_{n+1,1} + a_{n1}C_{n1}$$

$$= 1(-1)^{1+1}M_{11} + 1(-1)^{2+1}M_{21} + \dots + 1(-1)^{n+1}M_{n+1,1} + 1(-1)^{n+1}M_{n1}$$

$$= \begin{vmatrix} 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 1 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{vmatrix} + (-1)^{n+1} \begin{vmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & \dots & 1 & 1 \end{vmatrix}$$

$$= 1 + (-1)^{n+1}$$

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**Question 1.** (5 marks) Only using elementary operations show that

$$\begin{vmatrix} 1 & x & x^2 & x^3 \\ a & 1 & x & x^2 \\ p & b & 1 & x \\ q & r & c & 1 \end{vmatrix} = (1-ax)(1-bx)(1-cx)$$

$$\begin{aligned} LHS &= -xR_2 + R_1 \rightarrow R_1 \\ &\quad -xR_3 + R_2 \rightarrow R_2 \\ &\quad -xR_4 + R_3 \rightarrow R_3 \end{aligned} \begin{vmatrix} 1-ax & 0 & 0 & 0 \\ a-px & 1-bx & 0 & 0 \\ p-qx & b-rx & 1-cx & 0 \\ q & r & c & 1 \end{vmatrix} = (1-ax)(1-bx)(1-cx) = RHS$$

**Question 2.** (5 marks) By first performing a cofactor expansion along the first column show that

$$\begin{vmatrix} 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 1 \end{vmatrix} = 1 + (-1)^{n+1}$$