Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S7: Fall 2022: **Quiz 7** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Only using elementary operations show that

$$\begin{vmatrix} 1 & x & x^2 & x^3 \\ a & 1 & x & x^2 \\ p & b & 1 & x \\ q & r & c & 1 \end{vmatrix} = (1-ax)(1-bx)(1-cx)$$

$$\begin{vmatrix} 1 & x & x^2 & x^3 \\ a & 1 & x & x^2 \\ p & b & 1 & x \\ q & r & c & 1 \end{vmatrix} \underset{\substack{-aR_1+R_2\to R_2 \\ -pR_1+R_3\to R_3 \\ -qR_1+R_4\to R_4}}{=} \begin{vmatrix} 1 & x & x^2 & x^3 \\ 0 & 1-ax & x-ax^2 & x^2-ax^3 \\ 0 & b-px & 1-px^2 & x-px^3 \\ 0 & r-qx & c-qx^2 & 1-qx^3 \end{vmatrix}$$

Case 1: $(1-ax)=0$ then $LHS=0$ since R_2's entries are 0 and $RHS=0$

Case 2: $(1-ax)\neq 0$ then

$$\underset{\frac{1}{1-ax}R_2\to R_2}{=} (1-ax)\begin{vmatrix} 1 & x & x^2 & x^3 \\ 0 & 1 & x & x^2 \\ 0 & b-px & 1-px^2 & x-px^3 \\ 0 & r-qx & c-qx^2 & 1-qx^3 \end{vmatrix}$$

$$\underset{\substack{-(b-px)R_2+R_3\to R_3 \\ -(r-qx)R_2+R_4\to R_4}}{=} (1-ax)\begin{vmatrix} 1 & x & x^2 & x^3 \\ 0 & 1 & x & x^2 \\ 0 & 0 & 1-bx & x-bx^2 \\ 0 & 0 & c-r & 1-rx^2 \end{vmatrix}$$

Case 2a: $(1-bx)=0$ then $LHS=0$ since R_3's entries are 0 and $RHS=0$

Case 2b: $(1-bx)\neq 0$ then

$$\underset{\frac{1}{1-bx}R_3\to R_3}{=} (1-ax)(1-bx)\begin{vmatrix} 1 & x & x^2 & x^3 \\ 0 & 1 & x & x^2 \\ 0 & 0 & 1 & x \\ 0 & 0 & c-r & 1-rx^2 \end{vmatrix}$$

$$\underset{-(c-r)R_3+R_4\to R_4}{=} (1-ax)(1-bx)\begin{vmatrix} 1 & x & x^2 & x^3 \\ 0 & 1 & x & x^2 \\ 0 & 0 & 1 & x \\ 0 & 0 & 0 & 1-cx \end{vmatrix}$$

$$= (1-ax)(1+bx)(1-cx)$$

better solution on next page

Question 2. *(5 marks)* By first performing a cofactor expansion along the first column show that

$$\begin{vmatrix} 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 1 \end{vmatrix} = 1 + (-1)^{n+1}$$

$$LHS = a_{11}C_{11} + \underbrace{a_{21}C_{21} + \cdots + a_{n-1,1}C_{n-1,1}}_{0} + a_{n1}C_{n1}$$

$$= 1(-1)^{1+1}M_{11} + 1(-1)^{n+1}M_{n1}$$

$$= \begin{vmatrix} 1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{vmatrix} + (-1)^{n+1}\begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 1 \end{vmatrix}$$

$$= 1 + (-1)^{n+1}$$

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Question 1. *(5 marks)* Only using elementary operations show that

$$\begin{vmatrix} 1 & x & x^2 & x^3 \\ a & 1 & x & x^2 \\ p & b & 1 & x \\ q & r & c & 1 \end{vmatrix} = (1-ax)(1-bx)(1-cx)$$

$$LHS = \begin{matrix} -xR_2+R_1 \to R_1 \\ -xR_3+R_2 \to R_2 \\ -xR_4+R_3 \to R_3 \end{matrix} \begin{vmatrix} 1-ax & 0 & 0 & 0 \\ a-px & 1-bx & 0 & 0 \\ p-qx & b-rx & 1-cx & 0 \\ q & r & c & 1 \end{vmatrix} = (1-ax)(1-bx)(1-cx) = RHS$$

Question 2. *(5 marks)* By first performing a cofactor expansion along the first column show that

$$\begin{vmatrix} 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 1 \end{vmatrix} = 1+(-1)^{n+1}$$