

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531***. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. (5 marks) Given A , an $n \times n$ matrix such that $\det(A) = 9$ and

$$A^3 A^T = 3A^{-1} \text{adj}(A)$$

find n .

Question 2. (3 marks) Using Cramer's Rule find x_1 and x_3 for $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} \sin \theta & -\cos \theta & \sin \theta \\ \cos \theta & \sin \theta & \cos \theta \\ 0 & 0 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \sin \theta \\ 2 \cos \theta \\ 2 \end{bmatrix}$.

Question 3. Determine whether the following statements are true or false for any $n \times n$ matrices A and B . If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

1. (3 marks) There is no 3×3 matrix for which $A^2 + I_3 = 0$.

Bonus Questions. (5 marks) Show that the following two statements are equivalent:

S1. P , Q , and $P + Q$ are all invertible and $(P + Q)^{-1} = P^{-1} + Q^{-1}$

S2. P is invertible and $Q = PG$ where $G^2 + G + I = 0$.