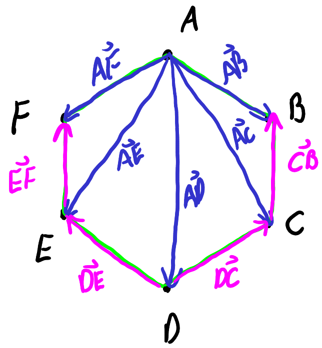
Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S7: Fall 2022: **Quiz 9** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Let A, B, C, D, E, and F be the vertices of a regular hexagon[1], taken in order. Show that $\vec{AB}+\vec{AC}+\vec{AD}+\vec{AE}+\vec{AF}=3\vec{AD}$



$\vec{AB}+\vec{AC}+\vec{AD}+\vec{AE}+\vec{AF}$

$=(\vec{AD}+\vec{DC}+\vec{CB})+(\vec{AD}+\vec{DC})+\vec{AD}+(\vec{AD}+\vec{DE})+(\vec{AD}+\vec{DE}+\vec{EF})$

$=3\vec{AD}+\vec{AD}+\vec{AD}+2\vec{DC}+\vec{CB}+2\vec{DE}+\vec{EF}$

note $\vec{AD}=\vec{AB}+\vec{BC}+\vec{CD}=\vec{AB}-\vec{CB}-\vec{DC}$

$\vec{AD}=\vec{AF}+\vec{FE}+\vec{ED}=\vec{AF}-\vec{EF}-\vec{DE}$

$=3\vec{AD}+(\vec{AB}-\cancel{\vec{CB}}-\cancel{\vec{DC}})+(\vec{AF}-\cancel{\vec{EF}}-\cancel{\vec{DE}})+\cancel{2}\vec{DC}+\cancel{\vec{CB}}+\cancel{2}\vec{DE}+\cancel{\vec{EF}}$

$=3\vec{AD}+\vec{AB}+\vec{AF}+\vec{DC}+\vec{DE}$

note $\vec{AB}=-\vec{DE}$

$\vec{AF}=-\vec{DC}$

$=3\vec{AD}+(-\vec{DE})+(-\vec{DC})+\vec{DC}+\vec{DE}$

$=3\vec{AD}$

Question 2. *(3 marks)* If $\vec{u}=(0,1,1)$ and $\vec{v}=(p,4,p)$ then find the parameter p such that the angle between $\vec{u}$ and $\vec{v}$ is $\pi/3$.

$\underline{u}\cdot\underline{v}=\|\underline{u}\|\|\underline{v}\|\cos\theta$

$(0,1,1)\cdot(p,4,p)=\|(0,1,1)\|\|(p,4,p)\|\cos\frac{\pi}{3}$

$4+p=\sqrt{2}\sqrt{p^2+4^2+p^2}\,\frac{1}{2}$

$8+2p=\sqrt{2}\sqrt{2p^2+16}$

$(8+2p)^2=2(2p^2+16)$

$64+32p+4p^2=4p^2+32$

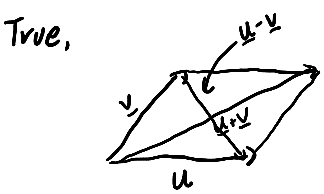
$32p=-32$

$p=-1$

Question 3. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

1. *(3 marks)* The diagonals of a rhombus[2] are perpendicular to each other.

True,



$(\underline{u}+\underline{v})\cdot(\underline{u}-\underline{v})=\underline{u}\cdot\underline{u}-\underline{u}\cdot\underline{v}+\underline{v}\cdot\underline{u}-\underline{v}\cdot\underline{v}$

$=\|\underline{u}\|^2-\underline{u}\cdot\underline{v}+\underline{u}\cdot\underline{v}-\|\underline{v}\|^2$

$=\|\underline{u}\|^2-\|\underline{v}\|^2$

$=0$ since $\|\underline{u}\|=\|\underline{v}\|$

[1] An *hexagon* is a closed geometrical shape with six sides and six angles. If an hexagon has equal sides and equal angles, then it is called a *regular hexagon.*

[2] A parallelogram with all sides of equal length is called a *rhombus*.