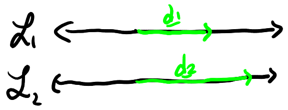
Dawson College: **Linear Algebra** (COMPUTER SCIENCE): 201-NYC-05-S9: Fall 2022: **Quiz 10** name: Y. Lamontagne

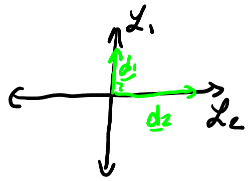
Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Given the lines $\mathbf{x} = (1,0,1) + t\overbrace{(1,1,-1)}^{d_1}$, $t \in \mathbb{R}$ and $\mathbf{x} = (2,3,4) + t\overbrace{(0,-1,2)}^{d_2}$, $t \in \mathbb{R}$..

a. *(2 marks)* Determine whether the lines are perpendicular to each other, parallel or neither. Justify.



The two lines are parallel iff their direction vector are parallel. Since $\underline{d_1}$ and $\underline{d_2}$ are not multiple of each other then the two lines are not parallel.



The two lines are perpendicular iff their direction vectors are perpendicular. Since $\underline{d_1} \cdot \underline{d_2} = (1,1,-1) \cdot (0,-1,2) = 1(0) + 1(-1) + (-1)(2) = -3 \neq 0$ the two lines are not perpendicular.

b. *(3 marks)* Find the point of intersection between the lines if it exists.

$\mathcal{L}_1$: $x = 1+t$, $y = t$, $z = 1-t$

$\mathcal{L}_2$: $x = 2$, $y = 3-s$, $z = 4+2s$

① $1 + t = 2$ $\Rightarrow t = 1$ sub into ②

② $t = 3 - s$

③ $1 - t = 4 + 2s$

$1 = 3 - s$

$s = 2$

Is it consistent? Check with ③

LHS $= 1 - 1 = 0$

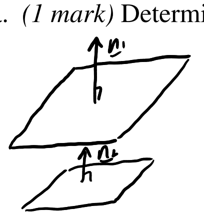
RHS $= 4 + 2(2) = 8 \neq$ LHS

$\therefore$ not consistent

$\therefore$ no intersection.

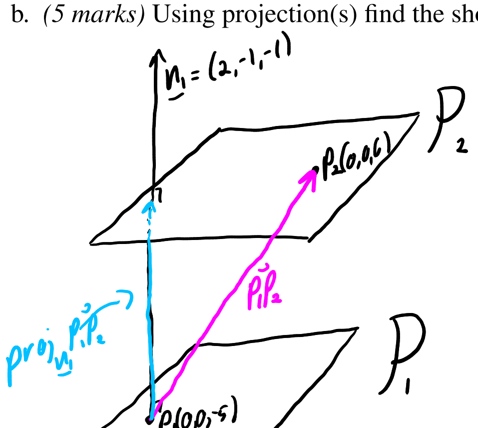
Questions 2. Given the planes $\overbrace{2x - y - z}^{\underline{n_1} = (2,-1,-1)} = 5$ and $\overbrace{-4x + 2y + 2z}^{\underline{n_2} = (-4,2,2)} = 12$.

a. *(1 mark)* Determine whether the two planes are perpendicular to each other, parallel or neither. Justify.



The two planes are parallel iff their normals are parallel. Since $\underline{n_2} = -2\underline{n_1}$ the two planes are parallel.

b. *(5 marks)* Using projection(s) find the shortest distance between the two planes.



Lets find a point on P_1. Let $x = y = 0$: $2(0) - 0 - z = 5$, $z = -5$ $\therefore P_1(0,0,-5)$

Lets find a point on P_2. Let $x = y = 0 \Rightarrow -4(0) + 2(0) + 2z = 12$, $z = 6$ $\therefore P_2(0,0,6)$

$\vec{P_1P_2} = \underline{P_2} - \underline{P_1} = (0,0,6) - (0,0,-5) = (0,0,11)$

$$\text{proj}_{\underline{n_1}} \vec{P_1P_2} = \frac{\underline{n_1} \cdot \vec{P_1P_2}}{\underline{n_1} \cdot \underline{n_1}} \underline{n_1}$$

$$= \frac{(2,-1,-1) \cdot (0,0,11)}{(2,-1,-1) \cdot (2,-1,-1)} (2,-1,-1)$$

$$= \frac{-11}{4+1+1} (2,-1,-1)$$

$$= \frac{-11}{6} (2,-1,-1)$$

$$\text{distance} = \| \text{proj}_{\underline{n_1}} \vec{P_1P_2} \|$$

$$= \left\| \frac{-11}{6} (2,-1,-1) \right\|$$

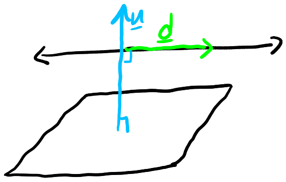
$$= \left| \frac{-11}{6} \right| \| (2,-1,-1) \|$$

$$= \frac{11}{6} \sqrt{2^2 + (-1)^2 + (-1)^2}$$

$$= \frac{11}{6} \sqrt{6}$$

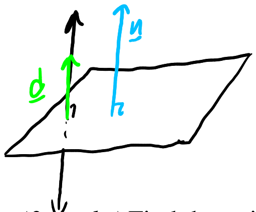
Questions 3. Given the plane $x+y+z=0$ and the line $(x,y,z)=(1+t,2+2t,3+3t)$ where $t\in\mathbb{R}$.

a. *(2 marks)* Determine whether the line is perpendicular to the plane, parallel or neither. Justify.



The line and the plane are parallel iff $\underline{n} \perp \underline{d}$.

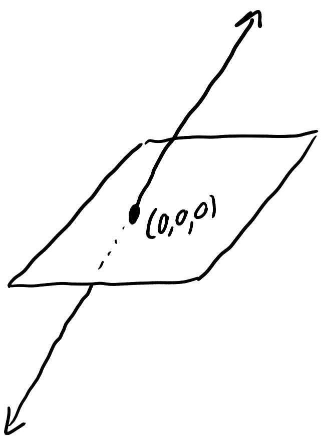
$\underline{n}\cdot\underline{d} = (1,1,1)\cdot(1,2,3) = 6 \neq 0$ $\therefore$ not parallel.



The line and the plane are orthogonal iff $\underline{n} \parallel \underline{d}$.

Since $\underline{n}$ and $\underline{d}$ are not multiples of each other, the line and the plane are not perpendicular.

b. *(3 marks)* Find the point of intersection between the line and the plane if it exists.



$$(1+t)+(2+2t)+(3+3t)=0$$
$$6+6t=0$$
$$t=-1$$

$$(x,y,z) = (1+(-1),\ 2+2(-1),\ 3+3(-1)) = (0,0,0)$$

$\therefore$ point of intersection is $(0,0,0)$.

Question Bonus. *(2 marks)* A former Prime Minister of Canada defined a proof as

> I don't know — a proof is a proof. What kind of a proof? It's a proof. A proof is a proof, and when you have a good proof, it's because it's proven.

In your own words correctly define proof.