Dawson College: **Linear Algebra** (COMPUTER SCIENCE): 201-NYC-05-S9: Fall 2022: **Quiz 12** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* The VECTOR Company compiles two project (Firecat (X) and Thunderfish (Y)) using two machines (A and B). Each compilation of Firecat requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each compilation of Thunderfish that is compiled requires 24 minutes processing time on machine A and 33 minutes processing time on machine B.
Available processing time for the week on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours.
Company policy is to maximise the combined sum of the compilations of Firecat and Thunderfishs by the end of the week.

a. (1 mark) Determine the objective function.

$Z = X + Y$

b. (3 marks) Find the constraints that define the feasible region.

$50X + 24Y \leq 40(60)$
$30X + 33Y \leq 35(60)$

$X \geq 0, \; Y \geq 0$

Questions 2. *(4 marks)* Consider the objective function $Z = 4x + 3y$ subject to the following constraints:

$$\begin{cases} x+y \geq 1 \\ x+2y \leq 4 \\ 2x+y \leq 4 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

If the vertices (corners) of the feasible region are $(1,0), (2,0), (0,2), (0,1), (4/3, 4/3)$. Find the maximum and minimum values of Z and at what (x,y) point they are reached. And justify!

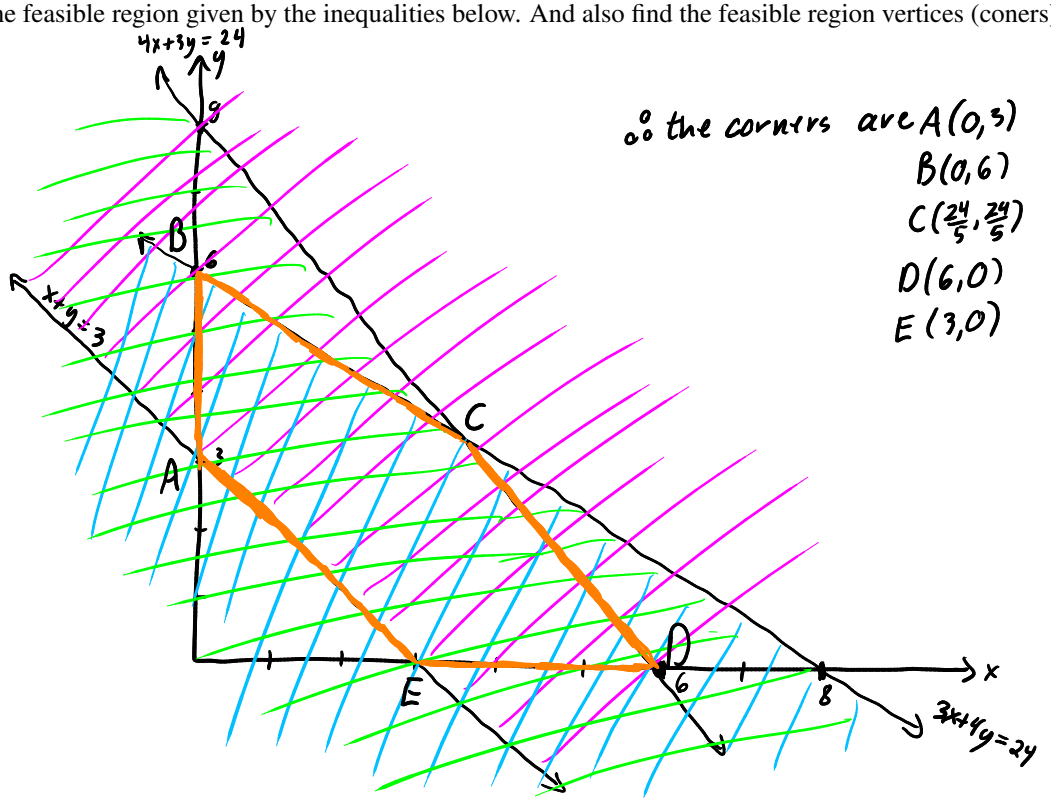
x	y	$Z = 4x+3y$
1	0	4
2	0	8
0	2	6
0	1	3
4/3	4/3	$4(\frac{4}{3}) + 3(\frac{4}{3}) = \frac{16}{3} + 4 = \frac{16}{3} + \frac{12}{3} = \frac{28}{3} \approx 9.3$

Minimum is 3 when $x=0$ and $y=1$
Maximum is $\frac{28}{3}$ when $x = y = \frac{4}{3}$

The above conclusion is true since linear objective function are optimized at the vertices (corners) of bounded feasible regions.

Questions 3. Sketch the feasible region given by the inequalities below. And also find the feasible region vertices (coners). Show your work!

$$\begin{cases} x+y \geq 3 \\ 3x+4y \leq 24 \\ 4x+3y \leq 24 \\ x \geq 0 \\ y \geq 0 \end{cases}$$



∴ the corners are A(0,3)
B(0,6)
$C(\frac{24}{5}, \frac{24}{5})$
D(6,0)
E(3,0)

To find C, lets determine the intersection between $3x+4y=24$ and $4x+3y=24$

$$\begin{bmatrix} 3 & 4 & 24 \\ 4 & 3 & 24 \end{bmatrix} \sim 3R_2 \to R_2 \begin{bmatrix} 3 & 4 & 24 \\ 12 & 9 & 72 \end{bmatrix}$$

$$\sim -4R_1+R_2 \to R_2 \begin{bmatrix} 3 & 4 & 24 \\ 0 & -7 & -24 \end{bmatrix}$$

$$\sim 7R_1 \to R_1 \begin{bmatrix} 21 & 28 & 168 \\ 0 & -7 & -24 \end{bmatrix}$$

$$\sim 4R_2+R_1 \to R_1 \begin{bmatrix} 21 & 0 & 72 \\ 0 & -7 & -24 \end{bmatrix}$$

$$\sim \begin{matrix} \frac{1}{21}R_1 \to R_1 \\ -\frac{1}{7}R_2 \to R_2 \end{matrix} \begin{bmatrix} 1 & 0 & 24/7 \\ 0 & 1 & 24/7 \end{bmatrix}$$

∴ $C(\frac{24}{7}, \frac{24}{7})$