

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531\*\*. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1. (10 marks)** Use the simplex method to solve the linear programming problem: Maximize the objective function:  $Z = 5x_1 + 2x_2 + 8x_3$  subject to

$$\begin{cases} 2x_1 - 4x_2 + x_3 \leq 42 \\ 2x_1 + 3x_2 - x_3 \leq 42 \\ 6x_1 - x_2 + 3x_3 \leq 42 \end{cases}$$

$$\begin{aligned} 2x_1 - 4x_2 + x_3 + s_1 &= 42 \\ 2x_1 + 3x_2 - x_3 + s_2 &= 42 \\ 6x_1 - x_2 + 3x_3 + s_3 &= 42 \\ -5x_1 - 2x_2 - 8x_3 + Z &= 0 \end{aligned}$$

Explicitly write the final value of the objective function, variables and slack variables.

	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$Z$	
	2	-4	1	1	0	0	0	42
	2	3	-1	0	1	0	0	42
	6	-1	3	0	0	1	0	42
	-5	-2	8	0	0	0	1	0
								$r_1 = 42/1 = 42$
								$r_3 = 42/3 = 14 \leftarrow \text{pivot row}$
	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$Z$	
	2	-4	1	1	0	0	0	42
	2	3	-1	0	1	0	0	42
$\frac{1}{3}R_3 \rightarrow R_3$	2	-1/3	1	0	0	1/3	0	14
	-5	-2	-8	0	0	0	1	0
	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$Z$	
	0	-11/3	0	1	0	-1/3	0	28
$-R_3 + R_1 \rightarrow R_1$	4	8/3	0	0	1	1/3	0	56
$R_3 + R_2 \rightarrow R_2$	2	-1/3	1	0	0	1/3	0	14
$8R_3 + R_4 \rightarrow R_4$	11	-14/3	0	0	0	8/3	1	112
	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$Z$	
	0	-11/3	0	1	0	-1/3	0	28
$\frac{3}{8}R_2 \rightarrow R_2$	3/2	1	0	0	3/8	1/8	0	21
	2	-1/3	1	0	0	1/3	0	14
	11	-14/3	0	0	0	8/3	1	112
	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$Z$	
	11/2	0	0	1	11/8	1/8	0	105
$14/3 R_2 + R_1 \rightarrow R_1$	3/2	1	0	0	3/8	1/8	0	21
$\frac{1}{3}R_2 + R_3 \rightarrow R_3$	5/2	0	1	0	1/8	3/8	0	21
$\frac{14}{3}R_2 + R_4 \rightarrow R_4$	18	0	0	0	7/4	13/4	1	210

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 21 \\ x_3 &= 21 \\ s_1 &= 105 \\ s_2 &= 0 \\ s_3 &= 0 \\ Z &= 210 \end{aligned}$$

**Bonus Question. (2 marks)** The barber is the "one who shaves all those, and those only, who do not shave themselves". The question is, does the barber shave himself?