

Question 1. (2 marks) Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If the number of equations in a linear system exceeds the number of unknowns, then the system must be inconsistent.

False, $x=1$
 $2x=2$ has more eqn. than var and has the solution $x=1$. Hence it is consistent

Question 2. (3 marks) Find all values of k for which the given augmented matrix corresponds to a consistent linear system.

$$\begin{bmatrix} 3 & -4 & k \\ -6 & 8 & 5 \end{bmatrix}$$

$$\begin{cases} 3x - 4y = k \\ -6x + 8y = 5 \end{cases}$$

$$\begin{cases} y = \frac{3}{4}x - \frac{k}{4} \\ y = \frac{3}{4}x + \frac{5}{8} \end{cases}$$

\therefore both lines are parallel since their slopes are equal. For the system to be consistent the y-intercept must be equal $\frac{-k}{4} = \frac{5}{8}$
 $k = -\frac{5}{2}$

Question 2. (3 marks) Given the linear system $\begin{cases} x - y + z = b_1 \\ 2x - 2y - 2z = b_2 \\ x + 3y - 5z = b_3 \end{cases}$. Determine the b_i if the linear system has the particular solution $(3, -2, 1)$.

The solution satisfies the equations $\therefore b_1 = 3 - (-2) + 1 = 6$
 $b_2 = 2(3) - 2(-2) - 2(1) = 8$
 $b_3 = 3 + 3(-2) - 5(1) = -8$

Question 3. (4 marks) Find the solution set of the linear equation by using parameters as necessary

$$3x_1 - 5x_2 + 4x_3 = 7$$

Also find two particular solutions.

Let $x_2 = s$
 $x_3 = t$ $s, t \in \mathbb{R}$

sub into eqn

$$3x_1 - 5s + 4t = 7$$

$$3x_1 = 7 + 5s - 4t$$

$$x_1 = \frac{7}{3} + \frac{5}{3}s - \frac{4}{3}t$$

$$\therefore x_1 = \frac{7}{3} + \frac{5}{3}s - \frac{4}{3}t$$

$$x_2 = s \quad s, t \in \mathbb{R}$$

$$x_3 = t$$

Particular sol.

1) $s = t = 0$

$$\therefore (x_1, x_2, x_3) = \left(\frac{7}{3}, 0, 0\right)$$

2) $s = 0, t = 1$

$$\therefore (x_1, x_2, x_3) = (1, 0, 1)$$