Dawson College: **Linear Algebra** (COMPUTER SCIENCE): 201-NYC-05-S9: Fall 2022: **Quiz 2** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(2 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If the reduced row echelon form of the augmented matrix for a linear system has a row of zeros, then the system must have infinitely many solutions.

False, $\begin{bmatrix}1&0&0\\0&0&1\\0&0&0\end{bmatrix}$ has no solution since $0x+0y=1$ cannot be satisfied by any x,y.

Question 2. *(3 marks)* The augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system.

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & b \\ \begin{bmatrix}1&2&0&-5&6\\0&0&1&-9&3\end{bmatrix} \end{array}$$

let $x_2 = s$, $x_4 = t$ $\quad s,t \in \mathbb{R}$

$\therefore x_1 = -2s+5t+6$, $x_2 = s$, $x_3 = 9t+3$, $x_4 = t$ $\quad s,t \in \mathbb{R}$

Question 3. *(5 marks)* Determine the values of p for which the system has no solutions, exactly one solution, or infinitely many solutions

$$\begin{array}{rcrcrcl} -x & + & 4y & - & 2z & = & 1 \\ -2x & + & 10y & + & (2p-4)z & = & 6 \\ 3x & - & 11y & + & (p^2+6)z & = & 5p-1 \end{array}$$

$$\begin{bmatrix}-1&4&-2&1\\-2&10&(2p-4)&6\\3&-11&(p^2+6)&5p-1\end{bmatrix}$$

$$\sim \begin{matrix}-2R_1+R_2\to R_2\\3R_1+R_3\to R_3\end{matrix} \begin{bmatrix}-1&4&-2&1\\0&2&2p&4\\0&1&p^2&5p-2\end{bmatrix}$$

$$\sim \tfrac{1}{2}R_2\to R_2 \begin{bmatrix}-1&4&-2&1\\0&1&p&2\\0&1&p^2&5p-2\end{bmatrix}$$

$$\sim -R_2+R_3\to R_3 \begin{bmatrix}-1&4&-2&1\\0&1&p&2\\0&0&p^2-p&5p\end{bmatrix}$$

unique solution: #leading entries in var. col. = #var

$p^2-p \neq 0$

$p(p-1) \neq 0$

$p \neq 0$ and $p-1 \neq 0$, $p \neq 1$

$\therefore p \neq 0, 1$

no solution: leading entry in constant col.

$p^2-p = 0$ and $5p \neq 0$

$p(p-1) = 0$, $p \neq 0$

$p=0$ or $p=1$

$\therefore p = 1$

infinitely many solutions: #leading entry < #var

$p^2-p = 0$ and $5p = 0$

$p(p-1) = 0$, $p = 0$

$p=0$ or $p=1$

$\therefore p = 0$