Dawson College: **Linear Algebra** (COMPUTER SCIENCE): 201-NYC-05-S9: Fall 2022: **Quiz 3** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(2 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If A and B are square matrices of the same order, then $(AB)^T = A^T B^T$.

False

Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ then $AB = \begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix}$ and $(AB)^T = \begin{bmatrix} 4 & 1 \\ 5 & 1 \end{bmatrix}$

But $A^T B^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 7 & 3 \end{bmatrix}$

Question 2. *(3 marks)* Prove: If A and B are $n \times n$ matrices, then $\text{tr}(A+B) = \text{tr}(A) + \text{tr}(B)$

Let $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix}$

LHS: $tr(A+B)$

$= tr\left(\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ a_{n1}+b_{n1} & a_{n2}+b_{n2} & \cdots & a_{nn}+b_{nn} \end{bmatrix}\right)$

$= a_{11}+b_{11}+a_{22}+b_{22}+\cdots+a_{nn}+b_{nn}$

$= a_{11}+a_{22}+\cdots+a_{nn}+b_{11}+b_{22}+\cdots+b_{nn}$

$= tr(A)+tr(B)$

$= RHS$

Question 3. *(5 marks)* Consider the matrices

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix},\ B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix},\ C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix},\ D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}.$$

Solve for X, if possible.

$\text{trace}(D)X + 2B = CA$

$5X + 2B = CA$ since $trace(D) = 1+0+4 = 5$

$5X = CA - 2B$

$5X = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} - 2\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$

$5X = \begin{bmatrix} 1 & 10 \\ 13 & 7 \end{bmatrix} - 2\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$

$5X = \begin{bmatrix} -7 & 12 \\ 13 & 3 \end{bmatrix}$

$X = \frac{1}{5}\begin{bmatrix} -7 & 12 \\ 13 & 3 \end{bmatrix}$