Dawson College: **Linear Algebra** (COMPUTER SCIENCE): 201-NYC-05-S9: Fall 2022: **Quiz 4** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(2 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

The sum of two invertible matrices of the same size must be invertible.

False,

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, both are invertible matrices since $ad-bc = 1 \neq 0$.

But $A+B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ which is not invertible.

Question 2. *(3 marks)* Show that if a square matrix A satisfies the equation $A^2 + 2A + I = 0$, then A must be invertible. What is the inverse?

We are given $A^2 + 2A + I = 0$

$I = -A^2 - 2A$

which implies $I = A(-A-2I)$ and $I = (-A-2I)A$

$\therefore$ A is invertible and $A^{-1} = -A - 2I$

Question 3. *(5 marks)* Solve for X given that it satisfies

$$\left(2A + X^T\right)^{-1} = I$$

where

$$A = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$$

$\left(\left(2A + X^T\right)^{-1}\right)^{-1} = I^{-1}$

$2A + X^T = I$

$X^T = I - 2A$

$X^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$

$X^T = \begin{bmatrix} -1 & -6 \\ -2 & -3 \end{bmatrix}$

$X = \begin{bmatrix} -1 & -2 \\ -6 & -3 \end{bmatrix}$