Dawson College: **Linear Algebra** (COMPUTER SCIENCE): 201-NYC-05-S9: Fall 2022: **Quiz 6** name: ____________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Determine conditions on the b_i's, if any, in order to guarantee that the linear system is consistent. And discuss your result using the Equivalence Theorem.

$$\begin{cases} x_1 - 2x_2 - x_3 = b_1 \\ -4x_1 + 5x_2 + 2x_3 = b_2 \\ -4x_1 + 7x_2 + 4x_3 = b_3 \end{cases}$$

Since the RREF of the coefficient matrix is I (if we had continued with Gauss Jordan), it follows by the equivalence theorem that the system is consistent for all constants.

$$\begin{bmatrix} 1 & -2 & -1 & b_1 \\ -4 & 5 & 2 & b_2 \\ -4 & 7 & 4 & b_3 \end{bmatrix}$$

$$\sim \begin{matrix} 4R_1+R_2\to R_2 \\ 4R_1+R_3\to R_3 \end{matrix} \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & -3 & -2 & b_2+4b_1 \\ 0 & -1 & 0 & b_3+4b_1 \end{bmatrix}$$

$$\sim R_2 \Leftrightarrow R_3 \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & -1 & 0 & b_3+4b_1 \\ 0 & -3 & -2 & b_2+4b_1 \end{bmatrix}$$

$$\sim -3R_2+R_3\to R_3 \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & -1 & 0 & b_3+4b_1 \\ 0 & 0 & -2 & -3b_3+b_2-8b_1 \end{bmatrix}$$

$\therefore$ system is consistent forall $b_i \in \mathbb{R}$.

Quations 2. *(1 mark)* Create a skew-symmetric matrix (that is, $A^T = -A$) by substituting appropriate numbers for the x's.

$$\begin{bmatrix} 0 & x & x & x \\ 3 & 0 & x & x \\ 7 & -8 & 0 & x \\ 2 & -3 & 9 & 0 \end{bmatrix}$$

-3 -7 -2 8 3 -9

Question 3. *(4 marks)* Prove: If $A^TA = A$, then A is symmetric and $A = A^2$.

premise:

- $A = A^TA$

conclusion:

- A is symmetric
- $A^2 = A$

First we want to show $A^T = A$

$$\begin{aligned} LHS &= A^T \\ &= (A^TA)^T \\ &= A^T(A^T)^T \\ &= A^TA \\ &= A \\ &= RHS \end{aligned}$$

and

$$\begin{aligned} A^2 &= AA \\ &= A^TA \quad \text{since } A \text{ is symmetric} \\ &= A. \end{aligned}$$