Dawson College: **Linear Algebra** (COMPUTER SCIENCE): 201-NYC-05-S9: Fall 2022: **Quiz 9** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Let $A = (0,-2,4)$, $B = (5,-5,3)$, and $P = (k,k,k)$.

a. *(4 marks)* Find k when the vector from A to B is perpendicular to the vector from A to P.

b. *(4 marks)* Find a vector of length 3 oppositely directed to $\vec{AB}$.

a) $\vec{AB} = \underline{B} - \underline{A} = (5,-5,3) - (0,-2,4) = (5,-3,-1)$

$\vec{AP} = \underline{P} - \underline{A} = (k,k,k) - (0,-2,4) = (k, k+2, k-4)$

$0 = \vec{AB} \cdot \vec{AP}$

$0 = (5,-3,-1) \cdot (k, k+2, k-4)$

$0 = 5(k) + (-3)(k+2) + (-1)(k-4)$

$0 = 5k - 3k - 6 - k + 4$

$2 = k$

b) $\dfrac{-3}{\|\vec{AB}\|}\vec{AB} = \dfrac{-3}{\sqrt{5^2+(-3)^2+(-1)^2}}(5,-3,-1) = \dfrac{-3}{\sqrt{35}}(5,-3,-1)$

Question 2. *(4 marks)* Let $\|\vec{u}\| = 3$, $\vec{v}$ be a unit vector, and the angle between $\vec{u}$ and $\vec{v}$ be 60 degrees. Find $\|5\vec{u} - 7\vec{v}\|$

$\|5\underline{u} - 7\underline{v}\|^2 = (5\underline{u} - 7\underline{v}) \cdot (5\underline{u} - 7\underline{v})$

$= (5\underline{u}) \cdot (5\underline{u}) + (5\underline{u}) \cdot (-7\underline{v}) + (-7\underline{v}) \cdot (5\underline{u}) + (-7\underline{v}) \cdot (-7\underline{v})$

$= 25\underline{u} \cdot \underline{u} - 35\,\underline{u} \cdot \underline{v} - 35\underline{u} \cdot \underline{v} + 49\,\underline{v} \cdot \underline{v}$

$= 25\|\underline{u}\|^2 - 70\,\underline{u} \cdot \underline{v} + 49\|\underline{v}\|^2$

$= 25(3)^2 - 70\|\underline{u}\|\|\underline{v}\|\cos 60^\circ + 49(1)^2$

$= 25(9) + 49 - 70(3)(1)\cos 60^\circ$

$= 274 - 210\left(\frac{1}{2}\right)$

$= 169$

$\|5\underline{u} - 7\underline{v}\| = 13$