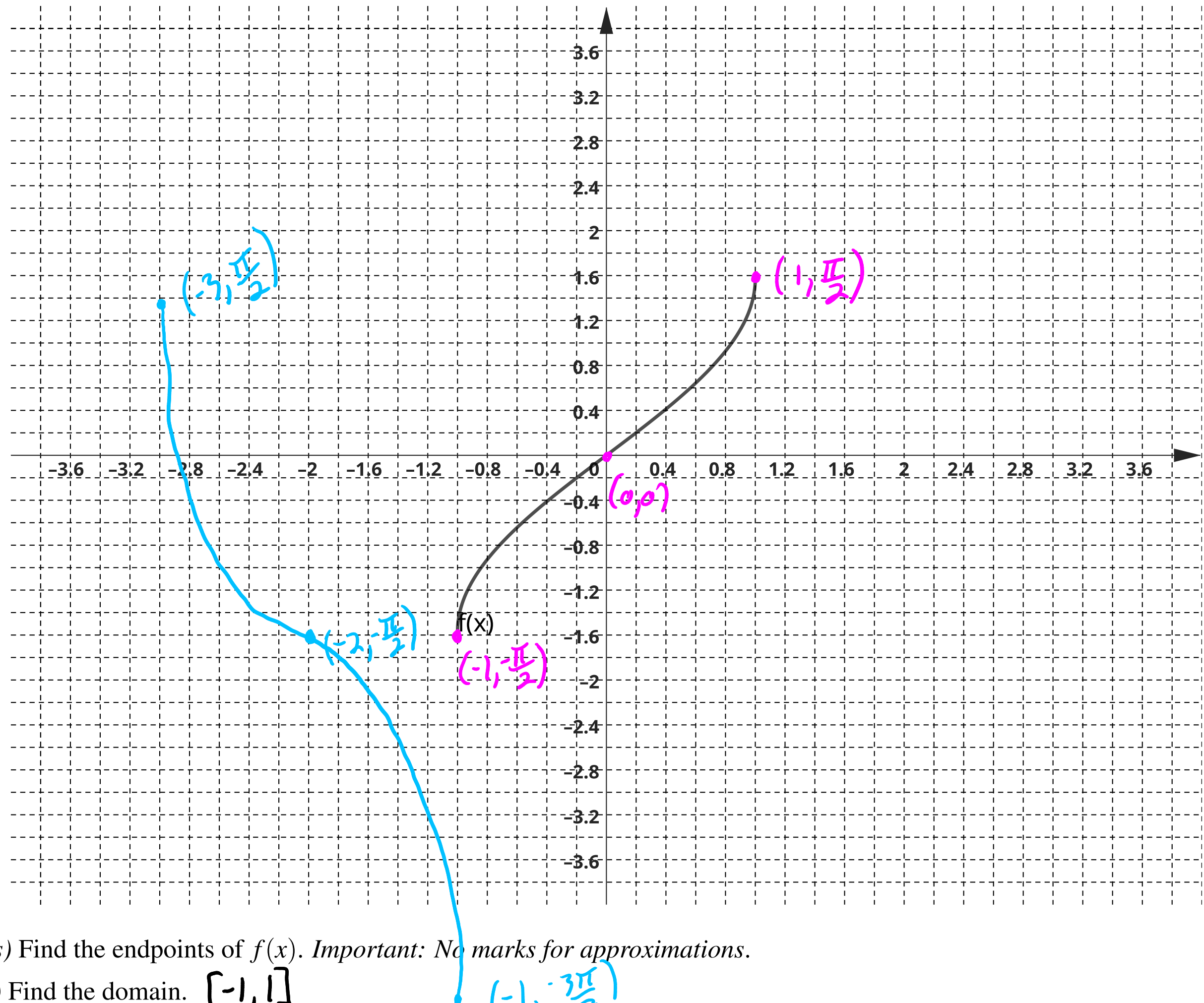
Dawson College: **Calculus I** (SCIENCE): 201-SN2-RE-S14: Fall 2024: **Quiz 1** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Given the graph of $f(x) = \arcsin(x)$.



a. *(2 marks)* Find the endpoints of $f(x)$. *Important: No marks for approximations.*

b. *(1 mark)* Find the domain. $[-1, 1]$ $(-1, -\frac{3\pi}{2})$

c. *(1 mark)* Find the range. $[-\frac{\pi}{2}, \frac{\pi}{2}]$

d. *(4 marks)* On the same graph as the above sketch $g(x) = -2f(x+2) - \frac{\pi}{2}$. Label the endpoints of the graph $g(x)$.

e. *(1 mark)* Find the limit

$$\lim_{x\to 0} f(x) = 0$$

if it exists.

f. *(1 mark)* Find the limit

$$\lim_{x\to 1^+} f(x)$$ Does not exists since f(x) is not defined for $x > 1$

if it exists.

Question 3. *(3 marks)* Find f and g where $h(x) = (f \circ g)(x) = \sqrt{\frac{x^2-1}{x-3}}$ and state the domain of h.

$f(g(x)) = \sqrt{\frac{x^2-1}{x-3}}$ For the domain $\frac{x^2-1}{x-3} \geq 0$ and $x \neq 3$

$\frac{(x-1)(x+1)}{x-3} \geq 0$

$\therefore$ $f(x) = \sqrt{x}$

$g(x) = \frac{x^2-1}{x-3}$

So the possibilities are ① $x-1 \leq 0$, $x+1 \leq 0$ and $x-3 > 0 \Rightarrow$ impossible

② $x-1 \geq 0$, $x+1 \geq 0$ and $x-3 > 0 \Rightarrow x > 3$

③ $x-1 \leq 0$, $x+1 \geq 0$ and $x-3 < 0 \Rightarrow -1 \leq x \leq 1$

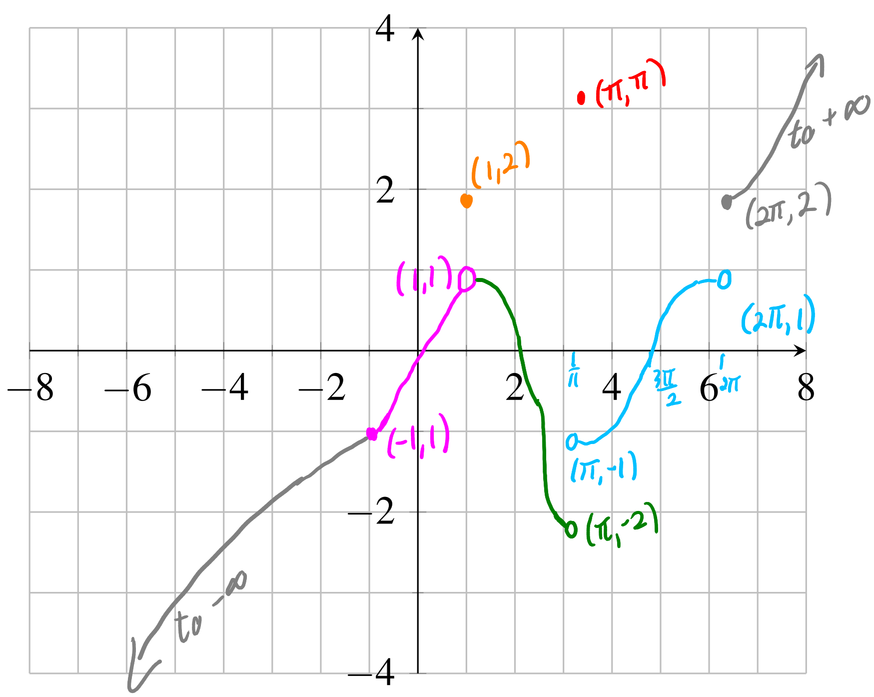
④ $x-1 \geq 0$, $x+1 \leq 0$ " $x-3 < 0 \Rightarrow 1 \leq x \leq -1$

$\therefore$ domain $[-1, 1] \cup (3, \infty)$

Question 3. Given the function $g(x)$ which has the real numbers as its domain and range. And satisfies all of the given conditions:

i. $g(x) = x$ if $x \in [-1, 1)$
ii. $g(x) = \cos x$ if $x \in (\pi, 2\pi)$
iii. $g(1) = 2$
iv. $g(\pi) = \pi$
v. $\lim_{x \to -1} g(x) = -1$
vi. $\lim_{x \to 1} g(x)$ exists
vii. $\lim_{x \to \pi^-} g(x) = -2$

a. *(4 marks)* Sketch $g(x)$, illustrate the behavior of the function on the entire domain, also appropriately label key points of the function.



b. *(2 marks)* Find all the value of $a \in (-1,\ 2\pi)$ for which

$\lim_{x \to a} g(x)$ does not exist, justify.

at $x = \pi$ since $\lim_{x \to \pi^-} g(x) \neq \lim_{x \to \pi^+} g(x)$

Question 4. *(4 marks)* Evaluate the difference quotient $\frac{f(a+h)-f(a)}{h}$ for $f(x) = \frac{x+2}{x+1}$. Simplify your answer.

$$= \frac{\frac{a+h+2}{a+h+1} - \frac{a+2}{a+1}}{h} = \frac{\frac{(a+h+2)(a+1)}{(a+h+1)(a+1)} - \frac{(a+2)(a+h+1)}{(a+h+1)(a+1)}}{h}$$

$$= \frac{\frac{\cancel{a^2} + \cancel{a} + \cancel{ah} + h + \cancel{2a} + \cancel{2} - [\cancel{a^2} + \cancel{ah} + \cancel{a} + \cancel{2a} + 2h + \cancel{2}]}{(a+h+1)(a+1)}}{h}$$

$$= \frac{\frac{-h}{(a+h+1)(a+1)}}{h}$$

$$= \frac{-\cancel{h}}{\cancel{h}(a+h+1)(a+1)} = \frac{-1}{(a+h+1)(a+1)}$$

Question 5 *(4 marks)* Guess the value of the limit (if it exists) $\lim_{h \to 0} \underbrace{\frac{(2+h)^2 - 32}{h}}_{f(h)}$ by evaluating the function at the numbers $\pm 0.01,\ \pm 0.001,\ \pm 0.0001$ (give the result correct to six decimal places).

h	$f(h)$
0.01	−2795.99
0.001	−27995.999
0.0001	−279995.9999

h	$f(h)$
−0.01	2803.99
−0.001	28003.999
−0.0001	280003.9999

$\lim_{h \to 0^-} f(h)$ DNE and $\lim_{h \to 0^+} f(h)$ DNE

$\therefore \lim_{h \to 0} f(h)$ DNE

Let $h(x) = f(g(x))$ where $f(x)$ and $g(x)$ are odd $\Rightarrow f(-x) = -f(x)$ and $g(-x) = -g(x)$
And we have that $h(-x) = f(g(-x)) = f(-g(x)) = -f(g(x)) = -h(x)$
$\therefore h(x)$ is an odd function.

Bonus. *(2 marks)* Show that the composition of two odd functions is an odd function.