Dawson College: **Calculus I** (SCIENCE): 201-SN2-RE-S14: Fall 2024: **Quiz 2** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If a function is not defined at a point, then it **might** have a jump discontinuity at that point.

b. If $|f(x)|$ is continuous at a, then $f(x)$ **might** also be continous at a.

c. If $f(x)$ is continuous at a, then $|f(x)|$ **must** also be continous at a.

Question 2. *(5 marks each)* Evaluate the following limits:

a. $\lim\limits_{x\to 4} \frac{x^3+12x^2+32x}{4x^2-64}$ $= \lim\limits_{x\to 4} \dfrac{x(x^2+12x+32)}{4(x^2-16)}$

$$= \lim_{x\to 4} \frac{x(x+8)\cancel{(x+4)}}{4(x-4)\cancel{(x+4)}}$$

$$= \lim_{x\to 4} \frac{x(x+4)}{4(x-4)} \quad \text{DNE}$$

since $\lim\limits_{x\to 4^-} \dfrac{x(x+4)}{4(x-4)} = -\infty$ and $\lim\limits_{x\to 4^+} \dfrac{x(x+4)}{4(x-4)} = \infty$

b. $\lim\limits_{x\to 0} f(x)$ where $f(x) = \begin{cases} x\cos\left(\frac{1}{x}\right) & \text{if } x<0 \\ -1+\sec x & \text{if } x \geq 0 \end{cases}$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} -1+\sec x = -1+\sec(0) = -1+1 = 0$ since $\sec x$ is continuous on its domain. In particular at $x=0$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} x\cos\left(\frac{1}{x}\right) = 0$ by squeeze thm.

Let $g(x) = x\cos\left(\frac{1}{x}\right)$

$K(x) = x \leq g(x) \leq -x = h(x)$ since the range of $\cos\left(\frac{1}{x}\right)$ is $[-1,1]$ and when $x \in (-\infty, 0)$

and $\lim\limits_{x\to 0^-} K(x) = \lim\limits_{x\to 0^-} -x = 0$

$\lim\limits_{x\to 0^-} h(x) = \lim\limits_{x\to 0^-} x = 0$

$\therefore \lim\limits_{x\to 0} f(x) = 0$ since $\lim\limits_{x\to 0^\pm} f(x) = 0$

c. $\lim\limits_{x\to 0} \frac{\sqrt{4+4\sin(2x^2)}-2}{8x^2}$ $\cdot \dfrac{\sqrt{4+4\sin(2x^2)}+2}{\sqrt{4+4\sin(2x^2)}+2}$

$$= \lim_{x\to 0} \frac{4+4\sin(2x^2)+\cancel{2\sqrt{4+4\sin(2x^2)}}-\cancel{2\sqrt{4+4\sin(2x^2)}}-4}{8x^2\left(\sqrt{4+4\sin(2x^2)}+2\right)}$$

$$= \lim_{x\to 0} \frac{4\sin(2x^2)}{8x^2\left(\sqrt{4+4\sin(2x^2)}+2\right)}$$

$$= \lim_{x\to 0} \frac{\sin(2x^2)}{2x^2\left(\sqrt{4+4\sin(2x^2)}+2\right)}$$

$$= \lim_{x\to 0} \frac{\sin(2x^2)}{2x^2} \lim_{x\to 0} \frac{1}{\sqrt{4+4\sin(2x^2)}+2}$$

$$= 1 \cdot \lim_{x\to 0} \frac{1}{\sqrt{4+4\sin(2x^2)}+2}$$

$$= \frac{\lim\limits_{x\to 0} 1}{\lim\limits_{x\to 0}\left(\sqrt{4+4\sin(2x^2)}+2\right)}$$

$$= \frac{1}{\lim\limits_{x\to 0}\sqrt{4+4\sin(2x^2)} + \lim\limits_{x\to 0} 2}$$

$$= \frac{1}{2+\sqrt{\lim\limits_{x\to 0}(4+4\sin(2x^2))}}$$

$$= \frac{1}{2+\sqrt{\lim\limits_{x\to 0} 4 + \lim\limits_{x\to 0} 4\sin(2x^2)}}$$

$$= \frac{1}{2+\sqrt{4+4\lim\limits_{x\to 0}\sin(2x^2)}}$$

$= \dfrac{1}{2+\sqrt{4}}$ since $\sin(2x^2)$ is continuous everywhere.

$= \dfrac{1}{4}$

$$= \lim_{x\to -1^-} \left[\frac{2}{-(x+1)} \left(\frac{1}{3} - \frac{1}{x+4} \right) \right]$$

$$= \lim_{x\to -1^-} \left[\frac{2}{-(x+1)} \left(\frac{x+4}{3(x+4)} - \frac{3}{3(x+4)} \right) \right]$$

$$= \lim_{x\to -1^-} \left[\frac{2}{-\cancel{(x+1)}} \frac{\cancel{(x+1)}}{3(x+4)} \right]$$

$$= \lim_{x\to -1^-} \frac{-2}{3(x+4)}$$

$= \dfrac{-2}{3(-1+4)}$ since the rational function is continuous at $x=-1$

$$= \frac{-2}{9}$$

$$\left(\frac{\sqrt{x^2+ax} + \sqrt{x^2+bx}}{\sqrt{x+ax} + \sqrt{x^2+bx}} \right)$$

$$= \lim_{x\to\infty} \frac{x^2+ax + \sqrt{x^2+ax}\sqrt{x^2+bx} - \sqrt{x^2+ax}\sqrt{x^2+bx} - (x^2+bx)}{\sqrt{x^2+ax} + \sqrt{x^2+bx}}$$

$$= \lim_{x\to\infty} \frac{ax-bx}{\sqrt{x^2(1+\frac{a}{x})} + \sqrt{x^2(1+\frac{b}{x})}}$$

$= \lim_{x\to\infty} \dfrac{x(a-b)}{|x|\sqrt{1+\frac{a}{x}} + |x|\sqrt{1+\frac{b}{x}}}$ $\quad |x| = x$ since $x\to\infty$

$$= \lim_{x\to\infty} \frac{\cancel{x}(a-b)}{\cancel{x}\left(\sqrt{1+\frac{a}{x}}^{\to 0} + \sqrt{1+\frac{b}{x}}^{\to 0}\right)}$$

$$= \frac{a-b}{\sqrt{1}+\sqrt{1}} = \frac{a-b}{2}$$

* $3x$ and x^2 are continuous on $\mathbb{R}$, $\frac{x^2-4}{x+2}$ is continuous on its domain $\therefore$ discontinuous at $x=-2$

$\therefore f(x)$ is not continuous at $x=-2$

At $x=-1$

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} \dfrac{x^2-4}{x+2} = \dfrac{(-1)^2-4}{-1+2} = -3$ since $\dfrac{x^2-4}{x+2}$ is continuous at $x=-1$

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} 3x = 3(-1) = -3$ since *

$\therefore \lim_{x\to -1} f(x) = -3 = f(-1)$ $\therefore$ continuous at $x=-1$

At $x=2$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 3x = 3(2) = 6$ since * and $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x^2 = 2^2 = 4$ since *

$\therefore \lim_{x\to 2} f(x)$ DNE since $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

$\therefore f(x)$ is continuous on $\mathbb{R} \setminus \{-2, 2\}$