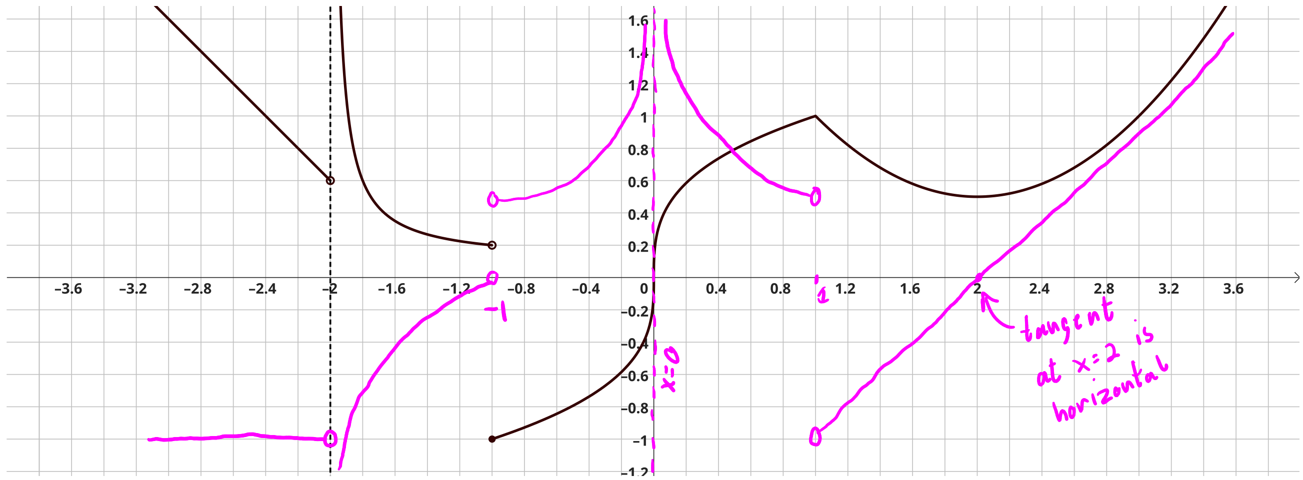
Dawson College: **Calculus I** (SCIENCE): 201-SN2-RE-S14: Fall 2024: **Quiz 3** name: Y.Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Given the graph of the function $f(x)$.



a. *(5 marks)* Sketch the graph of $f'(x)$ on the graph above. Label key points of your graph.

b. *(5 marks)* Identity the points where the function is not differentiable. Justify why it is not differentiable.

at $x=-2$ and -1 the function $f(x)$ is not continuous since $\lim_{x\to a} f(x)$ does not exist

$\therefore$ the function is not differentiable

at $x=0$ the function is not differentiable since $\lim_{x\to 0} f'(x)=\infty$

at $x=1$ the function is not differentiable at a corner (cusp) since

$$\lim_{h\to 0^-}\frac{f(x+h)-f(x)}{h} \neq \lim_{h\to 0^+}\frac{f(x+h)-f(x)}{h}$$

Question 2. Given the function $f(x)=1-\frac{2}{x}$

a. *(5 marks)* Is the function differentiable at $x=3$? Use the limit definition of the derivative.

b. *(3 marks)* Find the equation of the tangent line to the graph of $f(x)$ at $x=3$.

$$f'(3)=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$$

$$=\lim_{h\to 0}\frac{1-\frac{2}{3+h}-\left[1-\frac{2}{3}\right]}{h}$$

$$=\lim_{h\to 0}\frac{\frac{2}{3}-\frac{2}{3+h}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{2(3+h)}{3(3+h)}-\frac{2(3)}{3(3+h)}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{6+2h-6}{3(3+h)}}{h}$$

$$=\lim_{h\to 0}\frac{2\cancel{h}}{3\cancel{h}(3+h)}$$

$$=\lim_{h\to 0}\frac{2}{3(3+h)}=\frac{2}{9}$$

$m=\frac{2}{9}$

$y=mx+b$

The tangent at $x=3$ passes through $(x,f(x))$
$=(3,f(3))$
$=(3,\frac{1}{3})$

$\therefore\ \frac{1}{3}=\frac{2}{9}(3)+b$

$\frac{1}{3}=\frac{2}{3}+b$

$b=\frac{-1}{3}$

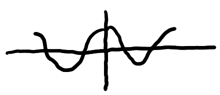
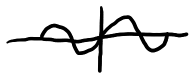
$\therefore$ equation of the tangent to $f(x)$ at $x=3$ is $y=\frac{2}{9}x-\frac{1}{3}$

Question 3. *(5 marks)* Find the value(s) of x for which the tangent to the graph of $f(x) = (\cos x)(\cos x) - 2\sin x$ is horizontal on the interval $[-2\pi, 2\pi]$.

$$f'(x) = (\cos x)'\cos x + \cos x(\cos x)' - 2\cos x$$
$$= \sin x \cos x + \cos x \sin x - 2\cos x$$
$$= -2\sin x \cos x - 2\cos x$$

$$0 = f'(x)$$
$$0 = -2\sin x \cos x - 2\cos x$$
$$0 = \sin x \cos x + \cos x$$
$$0 = \cos x(\sin x + 1)$$

$\cos x = 0$ $\qquad$ $\sin x + 1 = 0$
$\qquad\qquad\qquad$ $\sin x = -1$

at $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$ $\qquad$ at $x = -\frac{\pi}{2}, \frac{3\pi}{2}$

$\therefore$ the tangent is horizontal at $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$

Question 4. *(5 marks)* Find the derivative of $f(x) = \frac{\sqrt{x}\tan x}{x\sec x + \pi}$ but do not simplify.

$$f'(x) = \frac{(\sqrt{x}\tan x)'(x\sec x + \pi) - \sqrt{x}\tan x(x\sec x + \pi)'}{(x\sec x + \pi)^2}$$

$$= \frac{\left[\frac{1}{2\sqrt{x}}\tan x + \sqrt{x}\sec^2 x\right](x\sec x + \pi) - \sqrt{x}\tan x\left[\sec x + x\sec x\tan x\right]}{(x\sec x + \pi)^2}$$

Bonus Question. *(2 marks)* Find $\frac{d}{dx}|x^2 + x|$.