Dawson College: **Calculus I** (SCIENCE): 201-SN2-RE-S14: Fall 2024: **Quiz 4** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks each)* Find the following derivatives and **do not simplify**:

a. $\frac{d}{dx}\left[x\ln(\cos x) - \pi\sec^2(x^2+x+1)\right]$

$= \ln(\cos x) + x\frac{1}{\cos x}(-\sin x) - 2\pi\sec(x^2+x+1)(\sec(x^2+x+1))'$

$= \ln(\cos x) - x\tan x - 2\pi\sec(x^2+x+1)\sec(x^2+x+1)\tan(x^2+x+1)(x^2+x+1)'$

$= \ln(\cos x) - x\tan x - 2\pi\sec^2(x^2+x+1)\tan(x^2+x+1)(2x+1)$

b. $\frac{d}{dx}\left[\left(\frac{\sqrt{x}e^x}{\tan x}\right)^x\right]$ Let $y = \left(\frac{\sqrt{x}e^x}{\tan x}\right)^x$

$\ln y = \ln\left(\frac{\sqrt{x}e^x}{\tan x}\right)^x$

$\ln y = x\ln\left(\frac{\sqrt{x}e^x}{\tan x}\right)$

$\ln y = x\left[\ln\sqrt{x} + \ln e^x - \ln\tan x\right]$

$\ln y = x\left[\frac{1}{2}\ln x + x - \ln\tan x\right]$

$\frac{d}{dx}[\,"\,] = \frac{d}{dx}[\qquad]$

$\frac{1}{y}y' = \frac{1}{2}\ln x + x - \ln\tan x + x\left[\frac{1}{2x} + 1 - \frac{1}{\tan x}\sec^2 x\right]$

$y' = y\left[\ln\sqrt{x} - \ln\tan x + 2x + \frac{1}{2} - \frac{x\sec^2 x}{\tan x}\right]$

$y' = \left(\frac{\sqrt{x}e^x}{\tan x}\right)^x\left[\ln\left(\frac{\sqrt{x}}{\tan x}\right) + 2x + \frac{1}{2} - \frac{x\sec^2 x}{\tan x}\right]$

Question 2. *(5 marks)* Let $r(x) = f(g(h(x)))$, where $h(1) = 2$, $g(2) = 3$, $h'(1) = 4$, $g'(2) = 5$, and $f'(3) = 6$. Find $r'(1)$.

$r'(x) = f'(g(h(x)))(g(h(x)))'$

$= f'(g(h(x)))g'(h(x))h'(x)$

$r'(1) = f'(g(h(1)))g'(h(1))h'(1)$

$= f'(g(2))g'(2)(4)$

$= f'(3)\cdot 5\cdot 4$

$= 6\cdot 5\cdot 4$

$= 120$

Question 3. *(5 marks)* Given the curve $x^3 - y = 3x^2 - x + 4$, find all the point(s) on the curve where the tangent is parallel to the line $10x - y = 25$.

$$\frac{d}{dx}\left[x^3 - y\right] = \frac{d}{dx}\left[3x^2 - x + 4\right]$$

$$3x^2 - y' = 6x - 1$$

$$3x^2 - 6x + 1 = y'$$

$$10x - y = 25$$

$$y = 10x - 25$$

$\therefore$ slope of tangent is 10.

Lets determine when the slope of the tangent is 10.

$$10 = y'$$

$$10 = 3x^2 - 6x + 1$$

$$0 = 3x^2 - 6x - 9$$

$$0 = x^2 - 2x - 3$$

$$0 = (x-3)(x+1)$$

$x = 3$ $\quad$ $x = -1$

@ $x = 3$

$$(3)^3 - y = 3(3)^2 - 3 + 4$$

$$y = -1$$

$\therefore (3, -1)$

@ $x = -1$

$$(-1)^3 - y = 3(-1)^2 - (-1) + 4$$

$$y = -9$$

$\therefore (-1, -9)$

Question 4. *(5 marks)* For the curve $\ln x + xy = x - y$ find y' and y'' in terms of x and y but **do not simplify** y''.

$$\frac{d}{dx}\left[\ln x + xy\right] = \frac{d}{dx}\left[x - y\right]$$

$$\frac{1}{x} + y + xy' = 1 - y'$$

$$xy' + y' = 1 - \frac{1}{x} - y$$

$$(x+1)\,y' = \frac{x - 1 - xy}{x}$$

$$y' = \frac{\frac{x-1-xy}{x}}{x+1}$$

$$y' = \frac{x - 1 - xy}{x(x+1)}$$

$$\frac{d}{dx}\left[y'\right] = \frac{d}{dx}\left[\frac{x - 1 - xy}{x(x+1)}\right]$$

$$y'' = \frac{(1 - [y + xy'])(x^2 + x) - (x - 1 - xy)(2x + 1)}{(x(x+1))^2}$$

$$= \frac{\left(1 - y - \cancel{x}\left(\frac{x-1-xy}{\cancel{x}(x+1)}\right)\right)(x(x+1)) - \left[2x^2 - 2x - 2x^2y + x - 1 - xy\right]}{x^2(x^2+1)^2}$$

$$= \frac{((1-y)(x^2+x) - x\cancel{(x+1)}\left(\frac{x-1-xy}{\cancel{(x+1)}}\right) - 2x^2 + 2x + 2x^2y - x + 1 + xy}{x^2(x^2+1)^2}$$

$$= \frac{\cancel{x^2} + \cancel{x} - \cancel{yx^2} - \cancel{yx} - \cancel{x^2} + x - \cancel{x^2y} - 2x^2 + 2x + \cancel{2x^2y} \cancel{-x} + 1 + \cancel{xy}}{x^2(x^2+1)^2}$$

$$= \frac{-2x^2 + 3x + 1}{x^2(x^2+1)^2}$$