Dawson College: **Calculus I** (SCIENCE): 201-SN2-RE-S14: Fall 2024: **Quiz 5** name: Y. Lamontagne

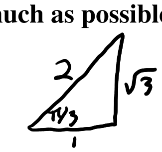
Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Evaluate:

a. *(3 marks)* $\lim_{x\to 0^+} \operatorname{arcsec}\left(1-\frac{1}{\ln x}\right)$ $= \operatorname{arcsec}\left(\lim_{x\to 0^+} 1 - \cancelto{0}{\frac{1}{\ln x}}\right) = \operatorname{arcsec} 1 = 0$

b. *(5 marks)* $g'(\pi/3)$ where $g(x) = x\arctan(\sin x)$ **(Simplify your answer as much as possible without using approximations.)**

$g'(x) = \arctan(\sin x) + \frac{x}{1+(\sin x)^2}\cos x$



$\sin\frac{\pi}{3} = \frac{opp}{hyp} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{3} = \frac{adj}{hyp} = \frac{1}{2}$

$\theta = \arctan\left(\frac{\sqrt{3}}{2}\right)$

$\tan\theta = \frac{\sqrt{3}}{2} = \frac{opp}{adj}$

$g'(\pi/3) = \arctan\left(\sin\frac{\pi}{3}\right) + \frac{\pi/3}{1+(\sin\frac{\pi}{3})^2}\cos\frac{\pi}{3}$

$= \arctan\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{3}\frac{1}{1+\left(\frac{\sqrt{3}}{2}\right)^2}\cdot\frac{1}{2}$

$= \arctan\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\frac{1}{1+\frac{3}{4}}$

$= \arctan\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{\cancel{6}\,3}\frac{\cancel{4}\,2}{4+3}$

$= \arctan\left(\frac{\sqrt{3}}{2}\right) + \frac{2\pi}{21}$

c. *(5 marks)* $\lim_{x\to\infty} (e^x+x)^{1/x}$

$y = \lim_{x\to\infty} (e^x+x)^{1/x}$ I.F. ∞^0

$\ln y = \ln \lim_{x\to\infty} (e^x+x)^{1/x}$

$\ln y = \lim_{x\to\infty} \ln(e^x+x)^{1/x}$

$\ln y = \lim_{x\to\infty} \frac{1}{x}\ln(e^x+x)$

$\ln y = \lim_{x\to\infty} \frac{\ln(e^x+x)}{x}$ I.F. $\frac{\infty}{\infty}$

$\ln y \overset{\hat{H}}{=} \lim_{x\to\infty} \frac{\frac{1}{e^x+x}\cdot(e^x+1)}{1}$

$\ln y = \lim_{x\to\infty} \frac{e^x+1}{e^x+x}$ I.F. $\frac{\infty}{\infty}$

$\ln y \overset{\hat{H}}{=} \lim_{x\to\infty} \frac{e^x}{e^x+x}$ I.F. $\frac{\infty}{\infty}$

$\ln y \overset{\hat{H}}{=} \lim_{x\to\infty} \frac{e^x}{e^x}$

$\ln y = \lim_{x\to\infty} 1$

$\ln y = 1$

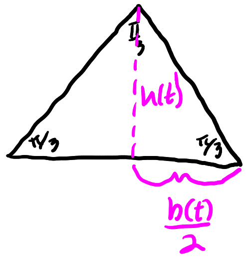
$e^{\ln y} = e^1$

$y = e$

$\therefore \lim_{x\to\infty} (e^x+x)^{1/x} = e$

Question 2. *(5 marks)* The sides of an equilateral triangle (a triangle with all sides equal) are increasing at the rate of 0.3 cm/s. At what rate is the area of the triangle changing when the side length is 5 cm?

$$\text{Area} = \frac{\text{base} \times \text{height}}{2}$$



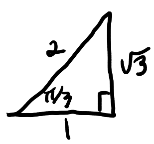
$A(t)$ - area triangle which depends on time
$b(t)$ - length of base which depends on time
$h(t)$ - heigth of triangle which depends on time

$$\tan\frac{\pi}{3} = \frac{\text{opp.}}{\text{adj.}}$$

$$\tan\frac{\pi}{3} = \frac{h(t)}{\frac{b(t)}{2}}$$

$$h(t) = \frac{b(t)}{2}\tan\frac{\pi}{3} = \frac{b(t)}{2}\frac{\sqrt{3}}{1}$$

$$= \frac{\sqrt{3}}{2}b(t)$$



$$\therefore A(t) = \left(b(t) \times \frac{\sqrt{3}}{2}b(t)\right)/2$$

$$= \frac{\sqrt{3}}{4}(b(t))^2$$

$$\frac{d}{dt}[A(t)] = \frac{d}{dt}\left[\frac{\sqrt{3}}{4}(b(t))^2\right]$$

$$A'(t) = \frac{\sqrt{3}}{4} \cdot 2\, b(t)\, b'(t)$$

$$= \frac{\sqrt{3}}{2}(5)(0.3)$$

$$= \frac{3\sqrt{3}}{4} \text{ cm}^2/\text{s}$$

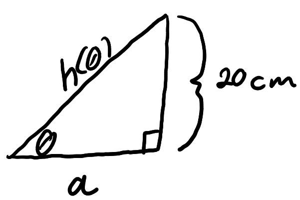
$$\doteq 1.3 \text{ cm}^2/\text{s}$$

Question 3. One side of a right triangle is known to be 20 cm long and the opposite angle is measured as 30°, with a possible error of ±1°.

a. *(3 marks)* Use differentials to estimate the error in computing the length of the hypotenuse.

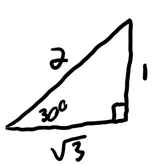
b. *(1 mark)* Explain why the error is negative when $\Delta\theta$ is positive.

c. *(1 mark)* Find an estimate of the percentage error using the differential?



$$\sin\theta = \frac{20}{h(\theta)}$$

$$h(\theta) = \frac{20}{\sin\theta} = 20\csc\theta$$



$$\csc 30^\circ = \frac{\text{hyp}}{\text{opp}} = \frac{2}{1}$$

$$\cot 30^\circ = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{3}}{1}$$

a) $\Delta h \approx dh$

$$= h'(\theta)d\theta$$

$$= \left(\frac{20}{\sin\theta}\right)' d\theta$$

$$= (20\csc\theta)' d\theta$$

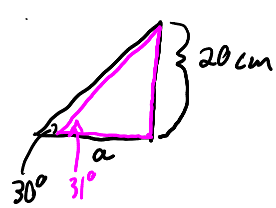
$$= (-20\csc\theta\cot\theta)\, d\theta$$

$$= -20\csc 30^\circ \cot 30^\circ \left(\pm\frac{\pi}{180}\right)$$

$$= \mp 20 \cdot 2\sqrt{3}\, \frac{\pi}{180}$$

$$= \mp\frac{2\sqrt{3}\pi}{9} \doteq -1.2 \text{ cm}$$

b)



If the opposite is fixed at 20cm then the adjacent and hypotenuse must diminish in length if the angle increases

c) $\left|\frac{\Delta h}{h}\right| \cdot 100\% \approx \left|\frac{dh}{h}\right| \cdot 100\%$

$$= \left|\frac{-20\csc\theta\cot\theta\, d\theta}{20\csc\theta}\right| 100\%$$

$$= \left|\cot 30^\circ\left(\pm\frac{\pi}{180}\right)\right| 100\%$$

$$= \frac{\sqrt{3}}{9}(5)\pi\,\% \doteq 3\%$$

Let $d\theta = \Delta\theta = \pm 1^\circ\left(\frac{\pi}{180}\right)$